



NICTA

Introduction

Framework

# Semantic Learning for Lazy Clause Generation

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- Finite Domain propagation
- Domains have a SAT encoding
- Propagators generate clauses explaining propagation
- SAT conflict analysis used to generate nogoods.

ex  $x + y \leq z : \llbracket x \geq 2 \rrbracket \wedge \llbracket z < 3 \rrbracket \rightarrow \llbracket y < 1 \rrbracket$

## SAT Domain Encoding

$$\llbracket x \leq d \rrbracket \rightarrow \llbracket x \leq d + 1 \rrbracket \text{ and}$$
$$\llbracket x = d \rrbracket \leftrightarrow (\llbracket x \leq d \rrbracket \wedge \neg \llbracket x \leq d - 1 \rrbracket)$$

- Possibly done lazily using previous / next existing literals.
- Generated for every domain update.

Given a set of conflict literal, resolve in trail order until a unique implication point is reach using the associated clauses.

## More general explanations:

- *clauses* are added to the SAT database
- *implication* are deleted on backtrack
- *backward* or *lazy* explanation are callbacks that generate the explanation during conflict analysis.

- does not exploit domain structure knowledge.
- does not handle sparse domain efficiently.
- generate unnecessary literals.

- Conflict analysis works on atomic explanations of the form  $\langle x \geq k \rangle$ ,  $\langle x = k \rangle$  and their negation  $\langle x < k \rangle$ ,  $\langle x \neq k \rangle$
- Literals used for unit propagation
- Domains associate explanation to atoms
- Explanations of the form : *none*, *implication*, *clause*, *backward*.

- SAT-style trail-ordered resolution
- Takes into account atom semantic
- Adding a bound atom in the conflict removes any implied atom from the conflict set

$\langle x < 2 \rangle \wedge \langle z = 1 \rangle \rightarrow \langle y < 2 \rangle :$

- $\{\langle y < 2 \rangle, \langle x! = 5 \rangle, l, \langle x! = 3 \rangle\}$ .
- $\{\langle y < 2 \rangle, \langle x < 4 \rangle, l\}$

- map maintains bounds updates with explanations
- map maintains existing literals

$\langle x \geq 2 \rangle, Exp \rightarrow \langle x \geq 5 \rangle:$

- fix literals  $\llbracket x \geq 4 \rrbracket$  to  $x \geq 5$  if they exist
- on untrailing : more literals might exist



- map maintains holes with explanation
- map maintains equality literals
- bound update : fixed any  $x = k$  literal in the range

$\langle x \geq 2 \rangle, Exp \rightarrow \langle x \geq 5 \rangle$ :

- fix literals  $\llbracket x \neq 3 \rrbracket$  to  $x \neq 4$  if they exist and are not already fixed.
- on backtrack, check  $Exp = exp(\llbracket x \neq k \rrbracket)$ !

$x > y: \langle x \geq 3 \rangle \rightarrow \langle y \geq 4 \rangle.$

$y > z: \langle y \geq 4 \rangle \rightarrow \langle z \geq 5 \rangle.$

$z \leq t: \langle t < 4 \rangle \wedge \langle z \geq 4 \rangle \rightarrow \text{false}$  (lifted conflict)

Explanations can be generated as clauses, as implications, or during conflict analysis:

- $z \leq t: \langle t < 4 \rangle \wedge \langle z \geq 4 \rangle \rightarrow \text{false}$
- $y > z: \langle y \geq 4 \rangle \rightarrow \langle z \geq 5 \rangle.$
- $x > y: \langle x \geq 3 \rangle \rightarrow \langle y \geq 4 \rangle.$

$x > y: \langle x \geq 3 \rangle \rightarrow \langle y \geq 4 \rangle.$   
 $y > z: \langle y \geq 4 \rangle \rightarrow \langle z \geq 5 \rangle.$   
 $z \leq t: \langle t < 4 \rangle \wedge \langle z \geq 4 \rangle \rightarrow \text{false}$

Backward explanation can be lifted using the atom semantic:

- $z \leq t: \langle t < 4 \rangle \wedge \langle z \geq 4 \rangle \rightarrow \text{false}$
- $y > z: \langle y \geq 3 \rangle \rightarrow \langle z \geq 4 \rangle.$
- $x > y: \langle x \geq 2 \rangle \rightarrow \langle y \geq 3 \rangle.$

Class	<i>n</i>	forward			backward			clausal		
		time	fails	len	time	fails	len	time	fails	len
amaze	(5)	113	272546	<b>16.8</b>	<b>96</b>	267012	18.2	455	<b>242110</b>	22.2
fast-food	(4)	345	241839	<b>44.3</b>	<b>264</b>	<b>214918</b>	45.6	>2617	58027 <sup>2</sup>	159.8
filters	(4)	<b>613</b>	<b>883948</b>	<b>11.2</b>	625	906724	20.7	>901	7331 <sup>1</sup>	10.0
league	(2)	11	74483	<b>28.3</b>	<b>10</b>	<b>72737</b>	31.1	14	81679	34.9
mssp	(6)	<b>23</b>	<b>55021</b>	<b>24.3</b>	29	62364	53.2	44	70511	24.6
nonogram	(4)	1965	96461	141.5	2124	90672	168.2	>3126	32805 <sup>2</sup>	144.8
pattern-set	(2)	451	<b>81397</b>	<b>180.4</b>	<b>400</b>	82410	180.8	>1016	3913 <sup>1</sup>	3505.3
proj-plan	(4)	83	<b>74531</b>	42.1	<b>78</b>	82269	63.4	150	89860	46.2
radiation	(2)	1.5	<b>7407</b>	<b>17.3</b>	jbf 1.3	7566	22.5	1.5	7382	19.9
ship-sched	(5)	43	44897	<b>16.0</b>	<b>37</b>	<b>41353</b>	18.2	273	71120	23.2
solbat	(5)	696	<b>337692</b>	<b>201.2</b>	<b>679</b>	357009	204.0	>1528	111477 <sup>1</sup>	239.1
still-life	(5)	735	<b>745949</b>	<b>21.9</b>	<b>678</b>	768155	30.2	>2640	269664 <sup>2</sup>	23.7
tp	(4)	613	8486	27.4	<b>126</b>	8490	30.1	>902	8330 <sup>1</sup>	12.7

Class	initial	root	forward # Lits	backward # Lits	clausal # Lits
amaze	8690	6409	<b>2214</b>	2231	3903
fast-food	1043606	729441	<b>9831</b>	11192	26373 <sup>2</sup>
filters	8204	6944	<b>1310</b>	1347	1285 <sup>1</sup>
league	341033	210981	<b>967</b>	969	1062
mssps	13534	9779	<b>6832</b>	6838	12373
nonogram	448437	364015	261867	262390	152213 <sup>2</sup>
pattern-set	19916	19795	<b>15490</b>	<b>15490</b>	14507 <sup>1</sup>
proj-plan	18720	18478	<b>5489</b>	6334	10612
radiation	145579	43144	<b>1993</b>	2071	7665
ship-sched	2070980	2070966	<b>12943</b>	13051	23370
solbat	12144	9326	10398	<b>10395</b>	8533 <sup>1</sup>
still-life	18947	12737	<b>3666</b>	3728	3964 <sup>2</sup>
tpp	19335	18976	9232	9253	7992 <sup>1</sup>

Class	backward				backward—no lifting			
	time	fails	lits	len	time	fails	lits	len
amaze	96	<b>267012</b>	11157	<b>18.2</b>	<b>94</b>	267482	<b>11138</b>	19.1
fast-food	<b>264</b>	<b>214918</b>	45771	45.6	306	241845	<b>44424</b>	47.2
filters	<b>625</b>	906724	5391	20.7	654	882678	<b>5376</b>	<b>20.6</b>
league	<b>10</b>	<b>72737</b>	<b>1938</b>	<b>31.1</b>	11	73549	1940	32.6
mssps	<b>29</b>	<b>62364</b>	41030	53.2	38	85741	<b>41028</b>	<b>44.4</b>
nonogram	2124	331186	<b>1049560</b>	168	<b>2122</b>	<b>315965</b>	1050193	<b>167</b>
pattern-set	<b>400</b>	<b>82410</b>	<b>30981</b>	<b>181</b>	403	<b>82410</b>	<b>30981</b>	<b>181</b>
proj-plan	<b>78</b>	82269	25336	<b>63.4</b>	79	<b>81666</b>	<b>25323</b>	71.1
radiation	2	<b>7566</b>	<b>4142</b>	<b>22.5</b>	<b>1</b>	7798	4145	22.6
ship-sched	<b>37</b>	<b>41353</b>	65258	18.2	41	46331	<b>65123</b>	<b>18.0</b>
solbat	679	<b>357009</b>	<b>51975</b>	<b>204</b>	668	<b>357009</b>	<b>51975</b>	<b>204</b>
still-life	<b>678</b>	768155	<b>18644</b>	30.3	768	<b>756075</b>	<b>18664</b>	<b>30.1</b>
tpp	<b>2827</b>	<b>342454</b>	<b>37014</b>	<b>30.1</b>	>3602	8492 <sup>1</sup>	24606	4.9