Explaining Propagators for Edge-valued Decision Diagrams

Graeme Gange[1] Peter J. Stuckey[1,2] Pascal Van Hentenryck[1,2]

National ICT Australia, Victoria Laboratory

Department of Computer Science and Software Engineering
The University of Melbourne, Vic. 3010, Australia
ggange@csse.unimelb.edu.au
{peter.stuckey,pvh}@nicta.com.au

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Outline

Finite-domain constraint satisfaction

Given a set of variables $\mathcal{V}$ with domains $\mathcal{D}$, and a set of constraints C over those variables, find an assignment to all variables in $\mathcal{V}$ such that all constraints in C are satisfied.

CP solvers interleave propagation (removing inconsistent values from domains) with backtracking search.

Lazy clause generation

A finite-domain CP engine with SAT-style conflict-directed learning.

- Lazily constructs a partial SAT model of the problem.

Complication:

- Propagators must be able to *explain* any inferences they make.

Motivation: Scheduling

We want to schedule n people among k tasks over m shifts.

- Each task requires a minimum coverage for each shift.
- There is a constraint on the set of legal shifts for each person.
- Each task has an associated cost.

The set of legal schedules can often be encoded as some form of automaton.

Scheduling constraints

$x_{ij} = d$ means person i does task d in shift j

$$
\begin{array}{lll}
\textbf{min} & cost & \\
\text{s.t.} & & \\
 & cost = \sum_{i,j} \text{taskcost}[x_{ij}] & \\
 & \texttt{regular}([x_{i1}, \ldots, x_{im}]) & \forall\ i \\
 & \sum_{i} (x_{ij} = d) \geq \text{cover}_{jd} & \forall\ j,\ d \\
 & x_{ij} \in \{1, \ldots, k\} & \forall\ i,\ j
\end{array}
$$

Unfortunately, this has extremely weak propagation between the schedules and the corresponding costs.

Basic model

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\begin{array}{ll}
\textbf{min} & \mathit{cost} \\
\text{s.t.} & \\
& \mathit{cost} = \sum_i c_i \\
& \texttt{cost\_regular}([x_{i1}, \ldots, x_{im}], c_i) \quad \forall\, i \\
& \sum_i (x_{ij} = d) \geq \text{cover}_{jd} \quad \forall\, j,\ d \\
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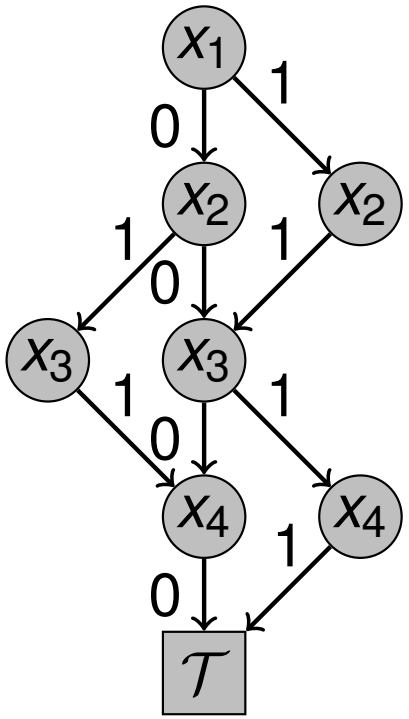
Multi-valued Decision Diagrams

- For a node $n = \langle x, [(v_1, n_1), \ldots, (v_k, n_k)] \rangle$:

  $\langle\langle n \rangle\rangle = (x = v_1 \wedge \langle\langle n_1 \rangle\rangle) \vee \ldots \vee (x = v_k \wedge \langle\langle n_k \rangle\rangle)$

- A path from the root to $\mathcal{T}$ represents a satisfying assignment.

How do we incorporate costs?

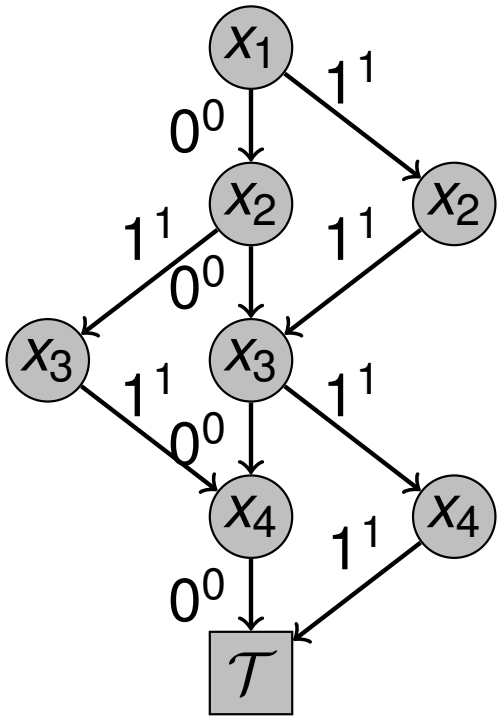


Edge-valued Decision Diagrams

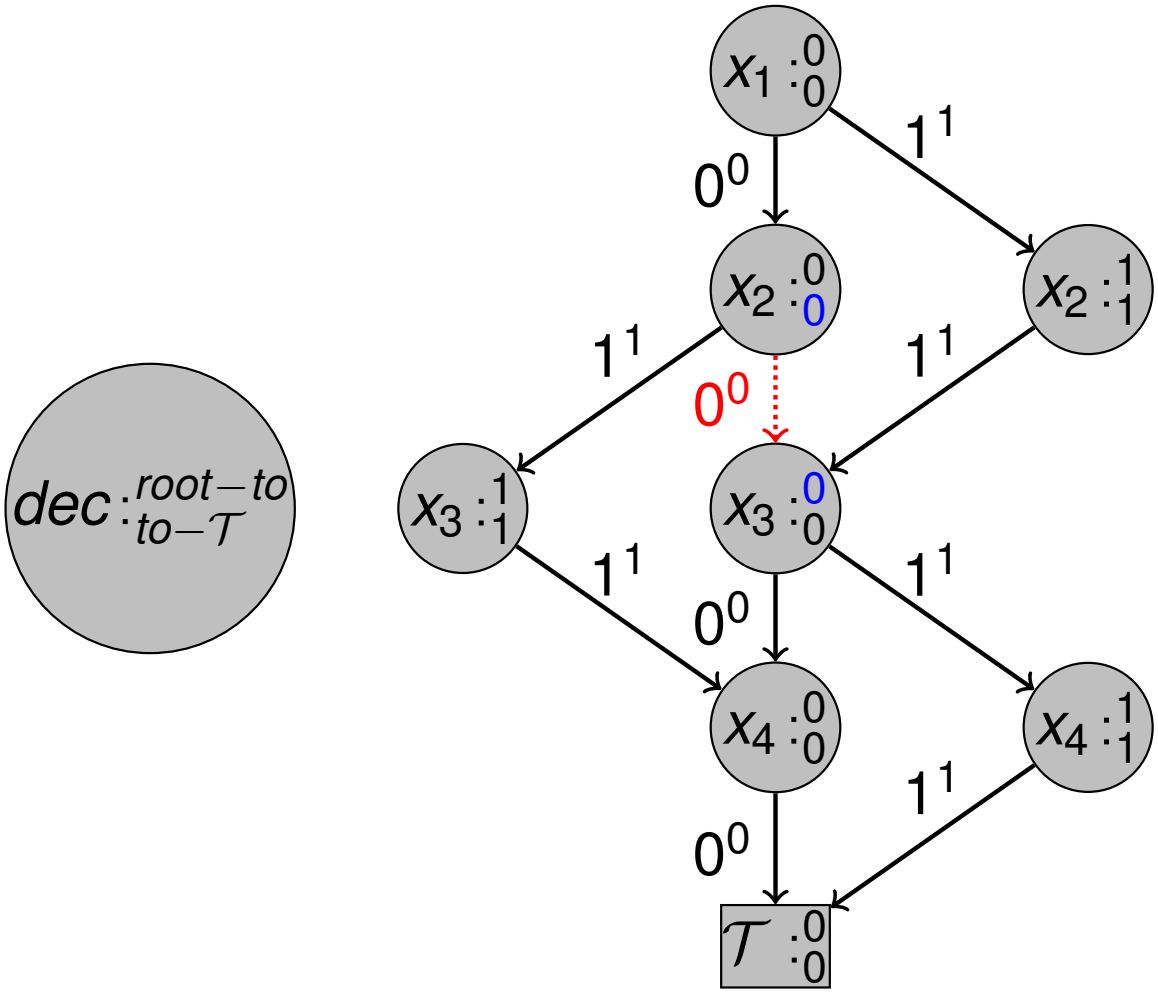
- Just like an MDD, but each edge has a corresponding cost w.
- For a node $n = \langle x, [(v_1, w_1, n_1), \ldots, (v_k, w_k, n_k)] \rangle$:

$$\langle\langle n \rangle\rangle = \min\{w_j + \langle\langle n_j \rangle\rangle \mid j \in 1..k, x_i = v_j\}$$

- Propagation is (conceptually) simple:
  - Maintain shortest path to/from each node.
  - When a value is removed from the domain, update the corresponding costs.
  - Eliminate any edges that only occur on infeasible paths.



Example: Propagating $[\![x_2 \neq 0]\!]$



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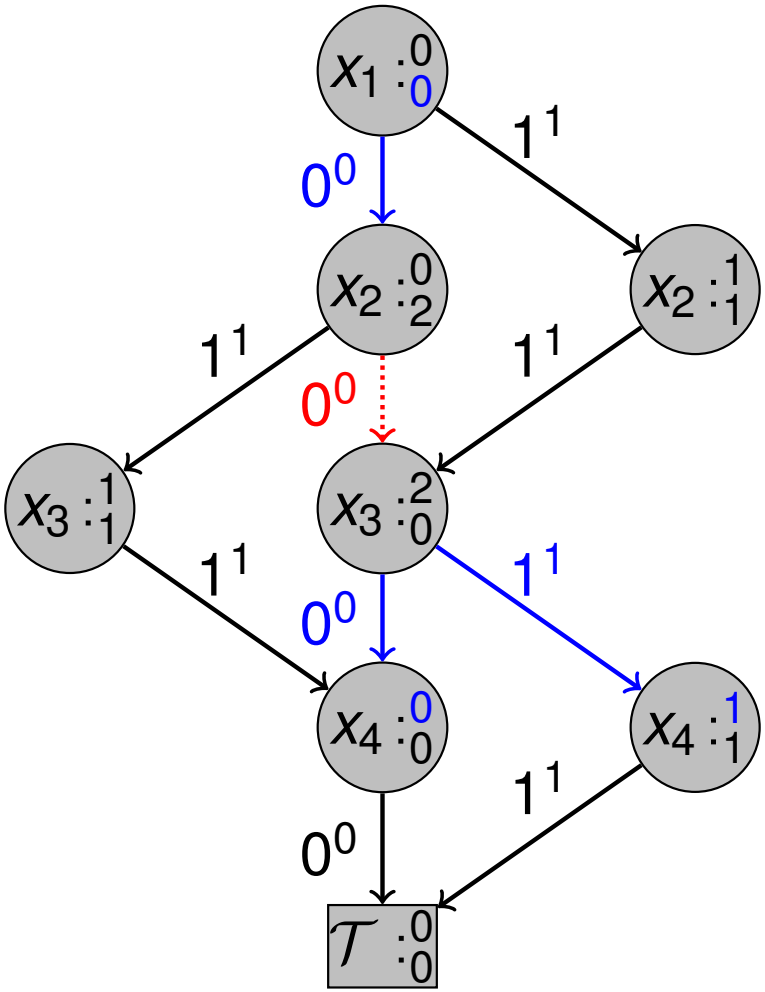

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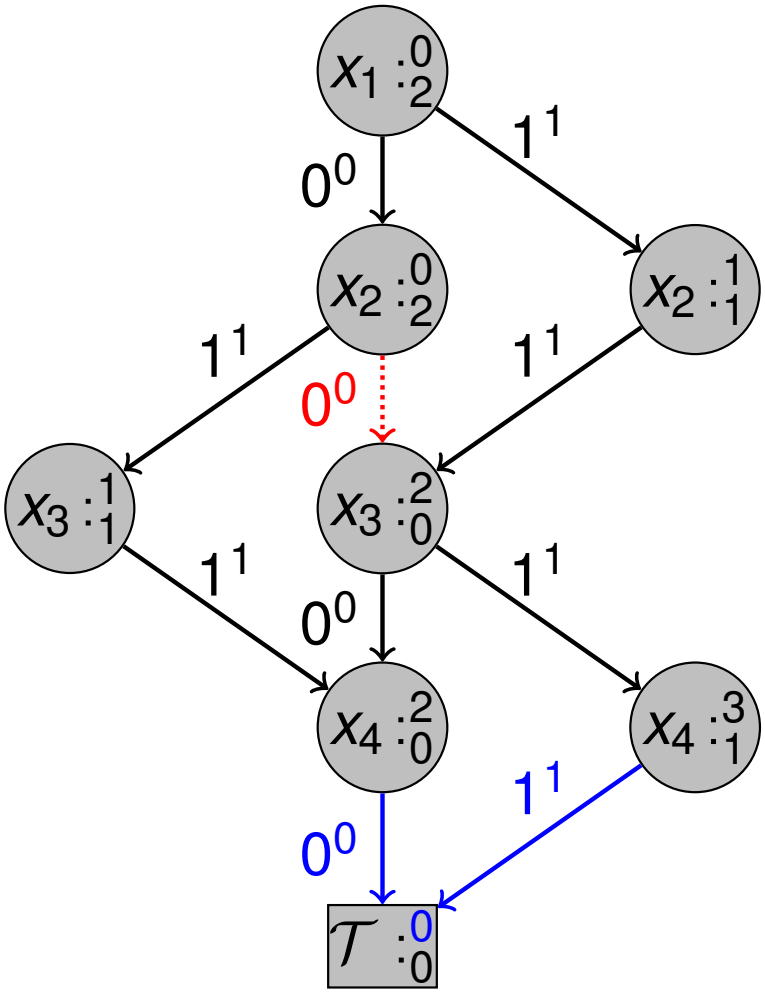

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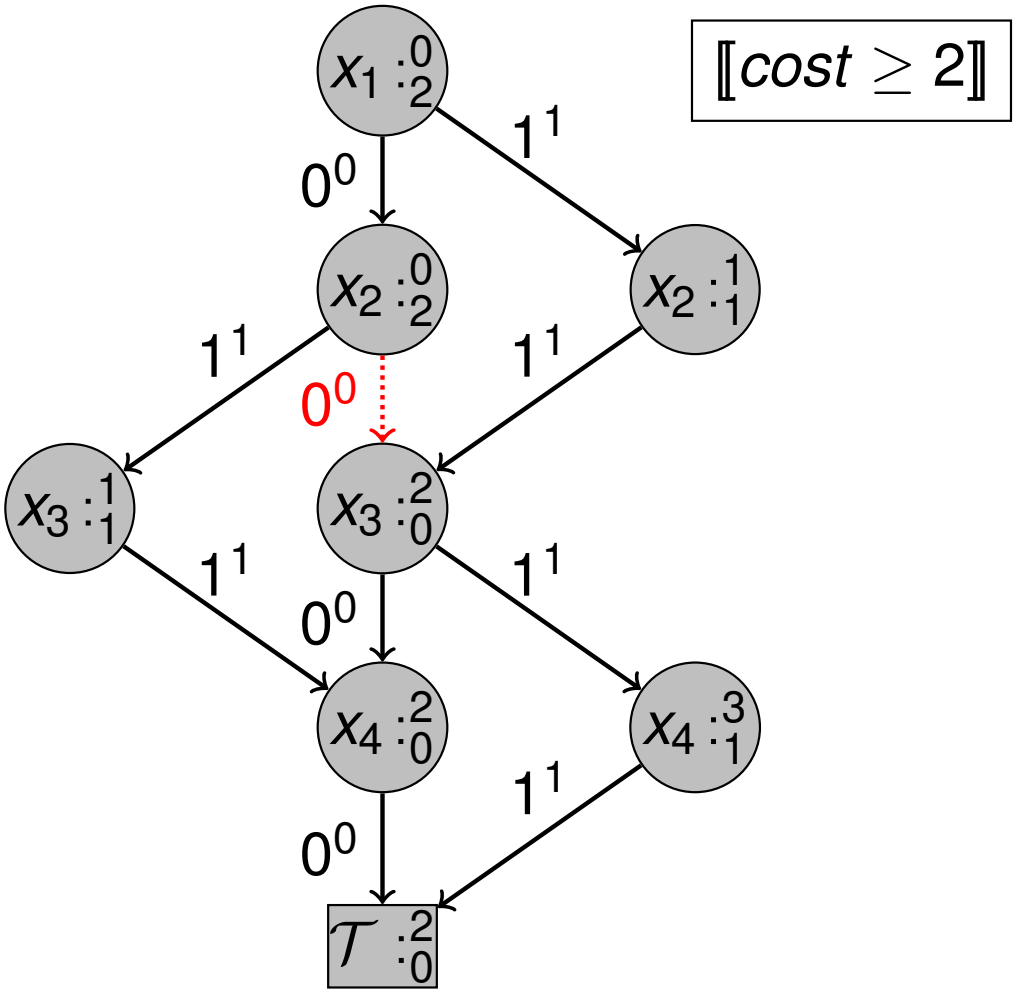


Propagating from ub(*cost*)

When ub(*cost*) changes, edges may become dead even though path lengths don't change.

- We don't want to scan the whole graph every time ub(*cost*) changes.
- Instead, keep a list of possibly-live edges for each value.
  - When ub(*cost*) changes, scan the list until we find a feasible edge.
  - Any examined infeasible edges are removed.
  - If no feasible edges are found, remove the value from the domain.

Explaining Inferences

Three kinds of inferences:

1. Infeasibility:

$$[\![cost \leq k]\!] \wedge [\![x_1 \neq v_1]\!] \wedge \ldots \wedge [\![x_m \neq v_m]\!] \models \bot$$

2. Variable domain:

$$[\![cost \leq k]\!] \wedge [\![x_1 \neq v_1]\!] \wedge \ldots \wedge [\![x_m \neq v_m]\!] \models [\![x \neq v]\!]$$

3. lb($cost$):

$$[\![x_1 \neq v_1]\!] \wedge \ldots \wedge [\![x_m \neq v_m]\!] \models [\![cost \geq k]\!]$$

Explaining Inferences

We can restate cases (2) and (3) in terms of feasibility:

1. Infeasibility:

$$[\![cost \leq k]\!] \wedge [\![x_1 \neq v_1]\!] \wedge \ldots \wedge [\![x_m \neq v_m]\!] \models \bot$$

2. Variable domain:

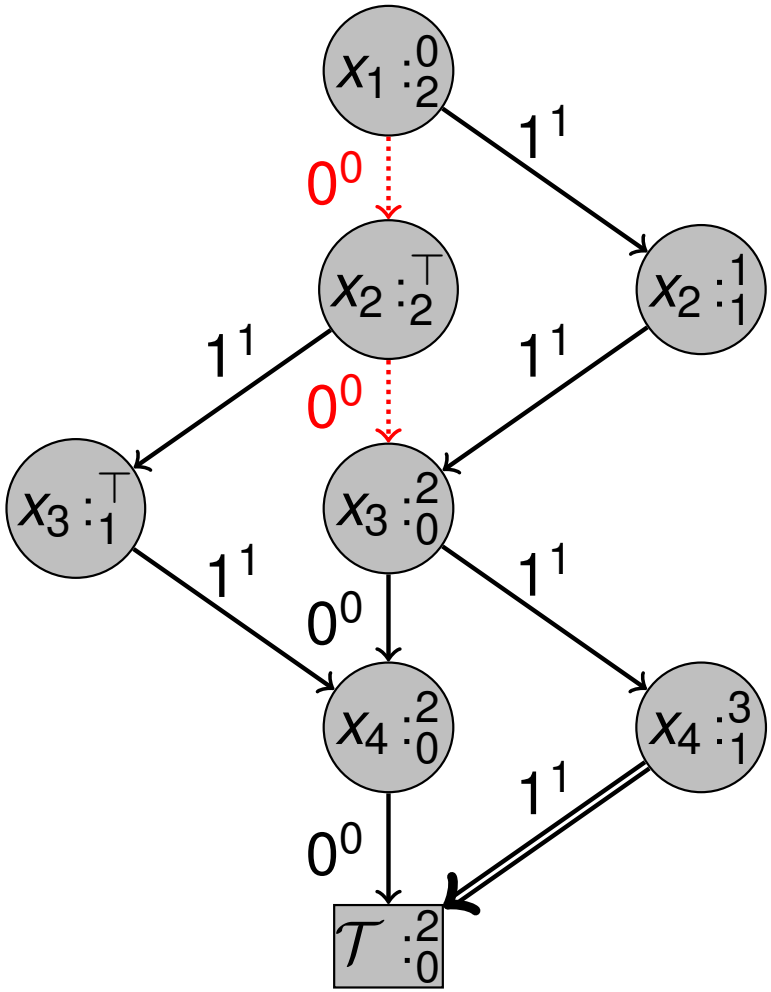
$$[\![cost \leq k]\!] \wedge [\![x_1 \neq v_1]\!] \wedge \ldots \wedge [\![x_m \neq v_m]\!] \wedge [\![x = v]\!] \models \bot$$

3. lb($cost$):

$$[\![x_1 \neq v_1]\!] \wedge \ldots \wedge [\![x_m \neq v_m]\!] \wedge [\![cost \leq k - 1]\!] \models \bot$$

Naive Explanation

- Always use current cost bound and current domain
- $[\![cost \leq 2]\!] \wedge [\![x_1 \neq 0]\!] \wedge [\![x_2 \neq 0]\!] \wedge [\![x_4 = 1]\!] \models \bot$
- Naive construction is valid.
- But produces terrible nogoods



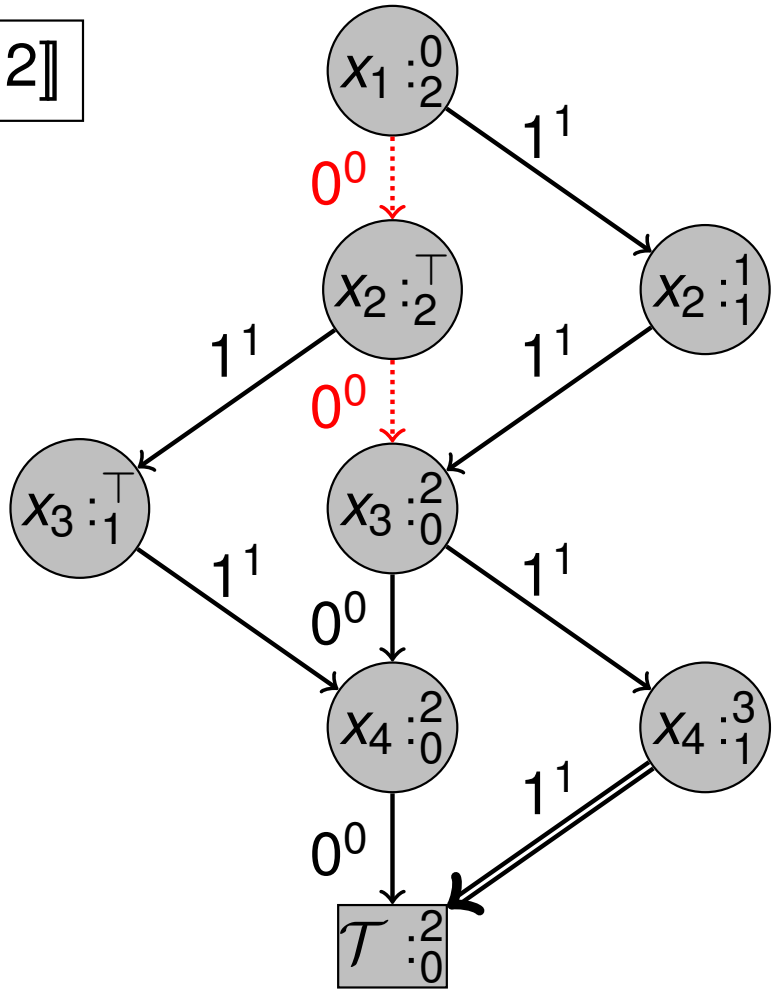
Minimal Explanation

Essentially a generalization of the MDD explanation algorithm.

- Compute the weakest possible cost bound k'
- Then progressively collect only those literals necessary to eliminate all paths of length $\leq k'$.

Example: Explaining $[\![x_4 \neq 1]\!]$

$[\![cost \leq 2]\!]$



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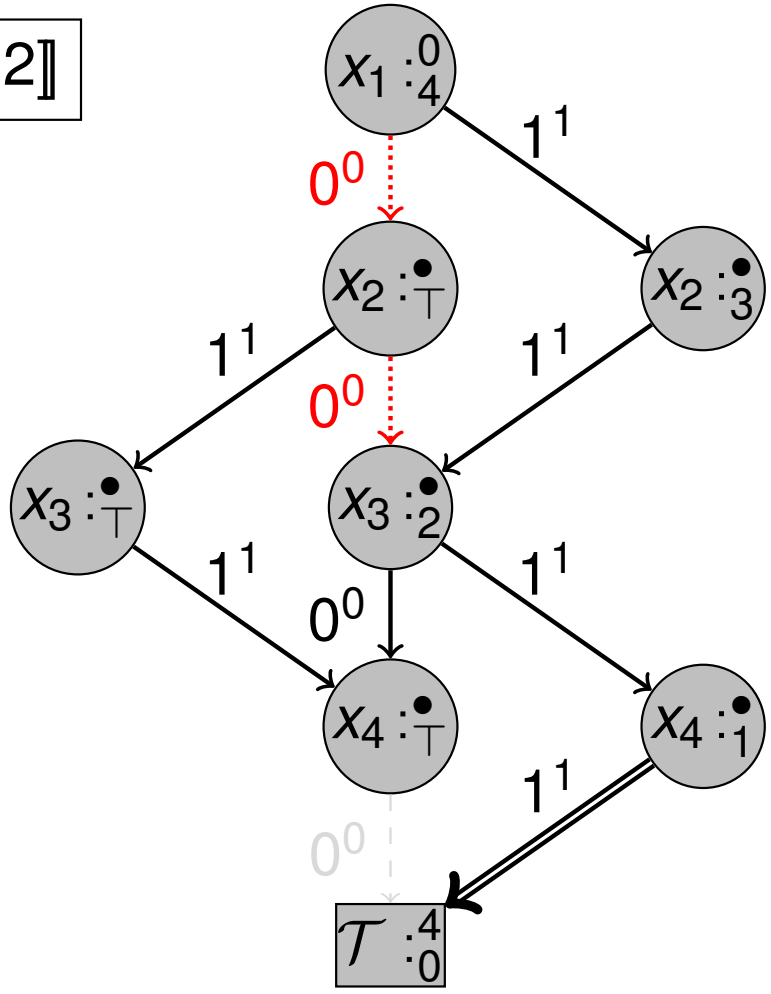

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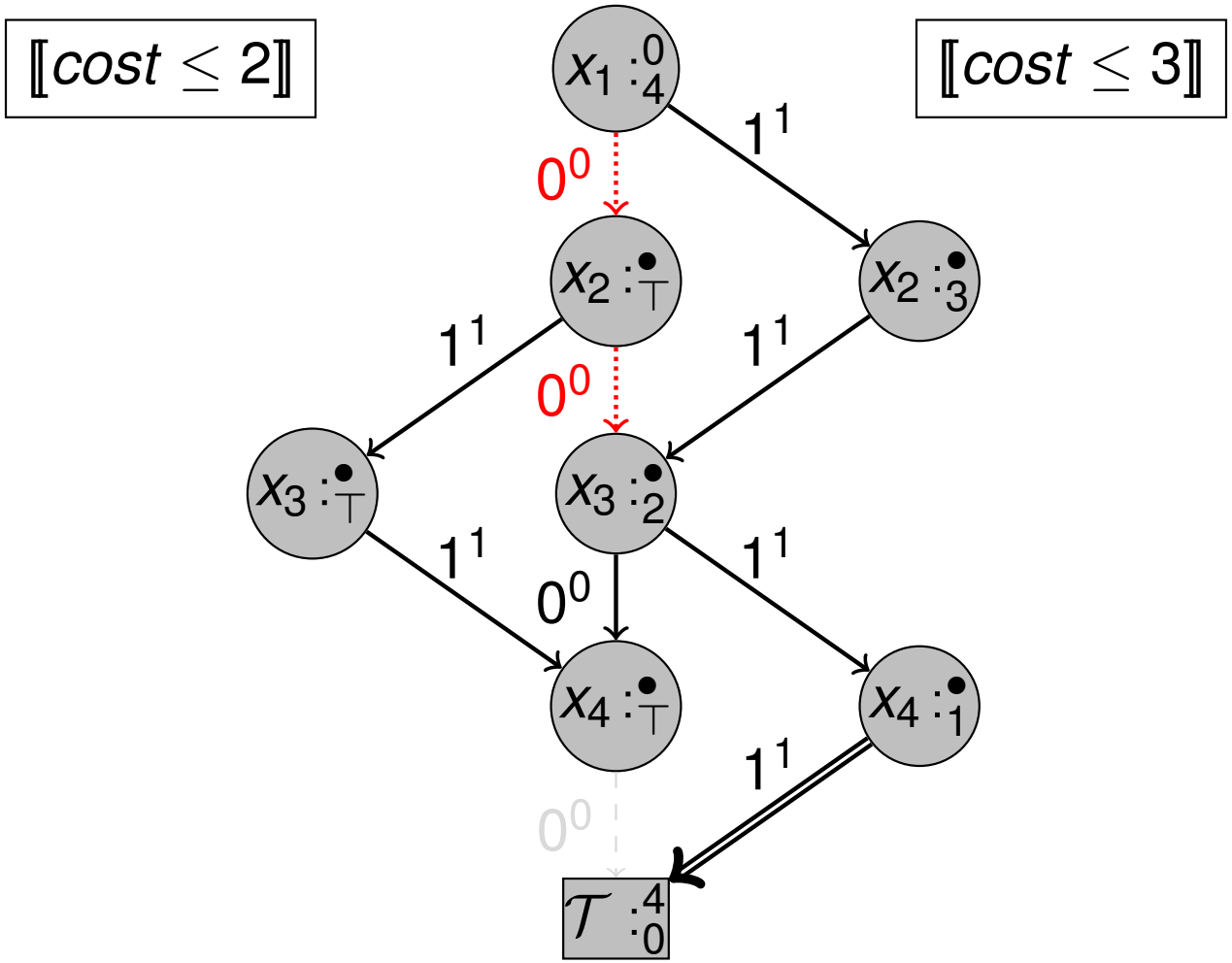

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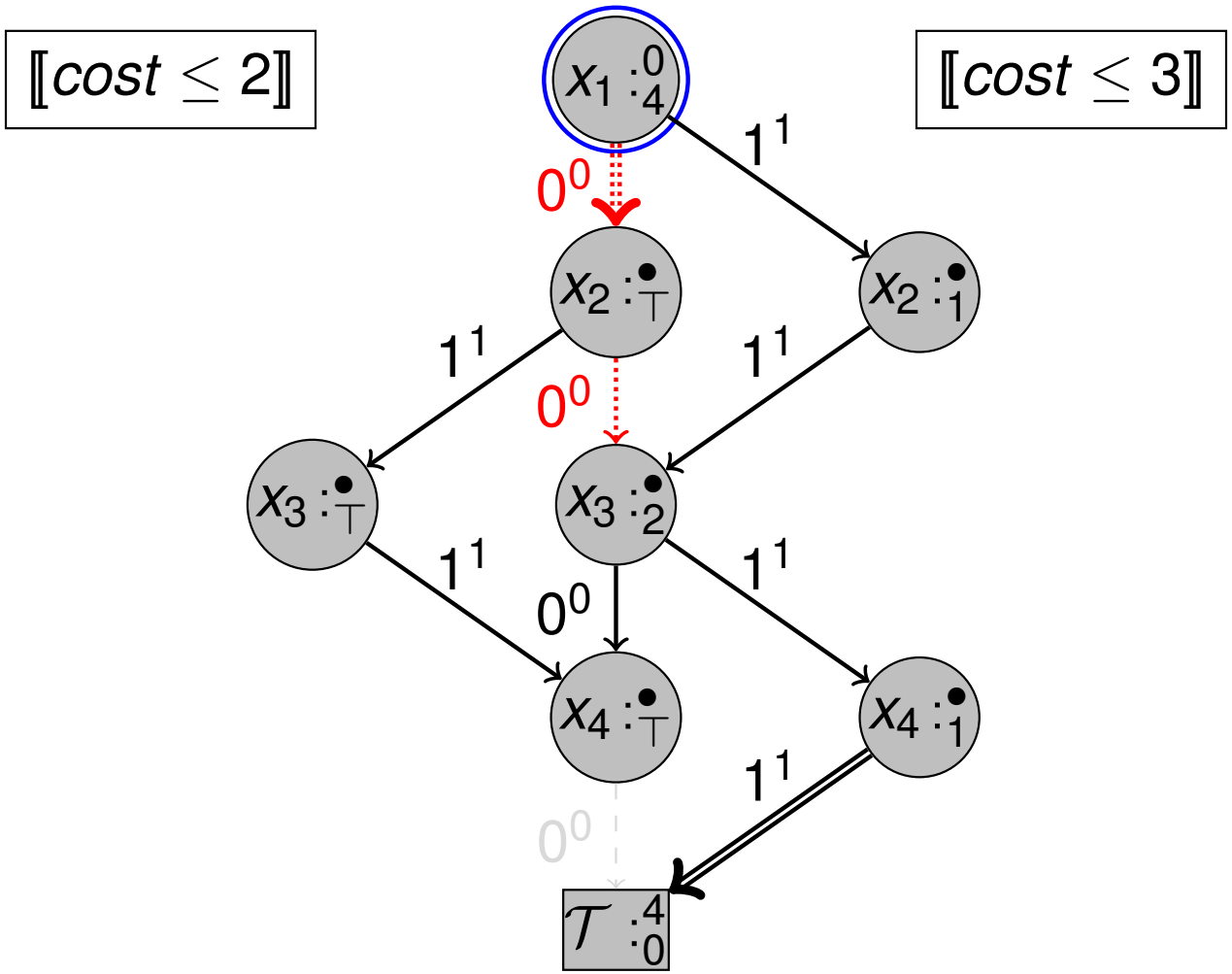

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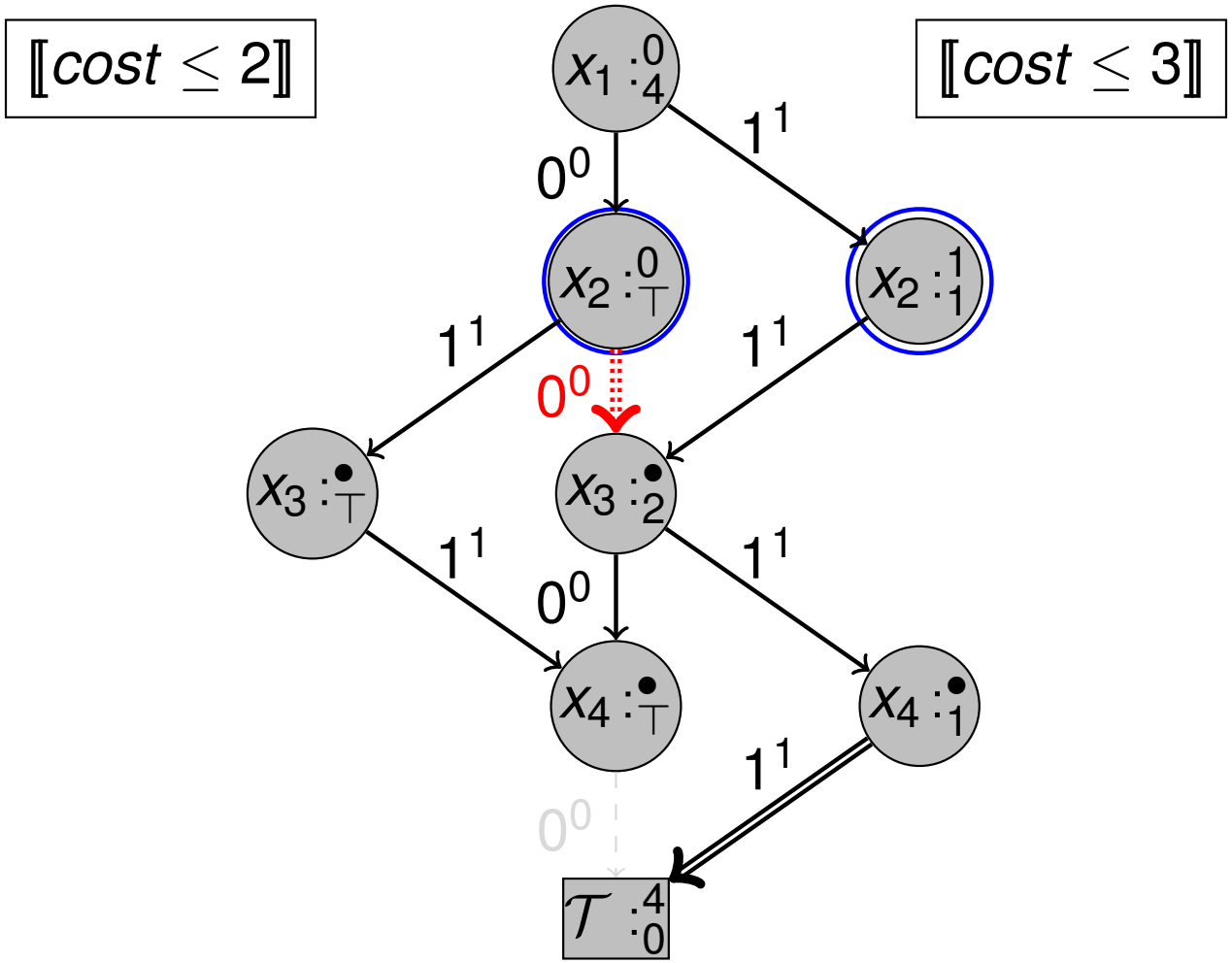

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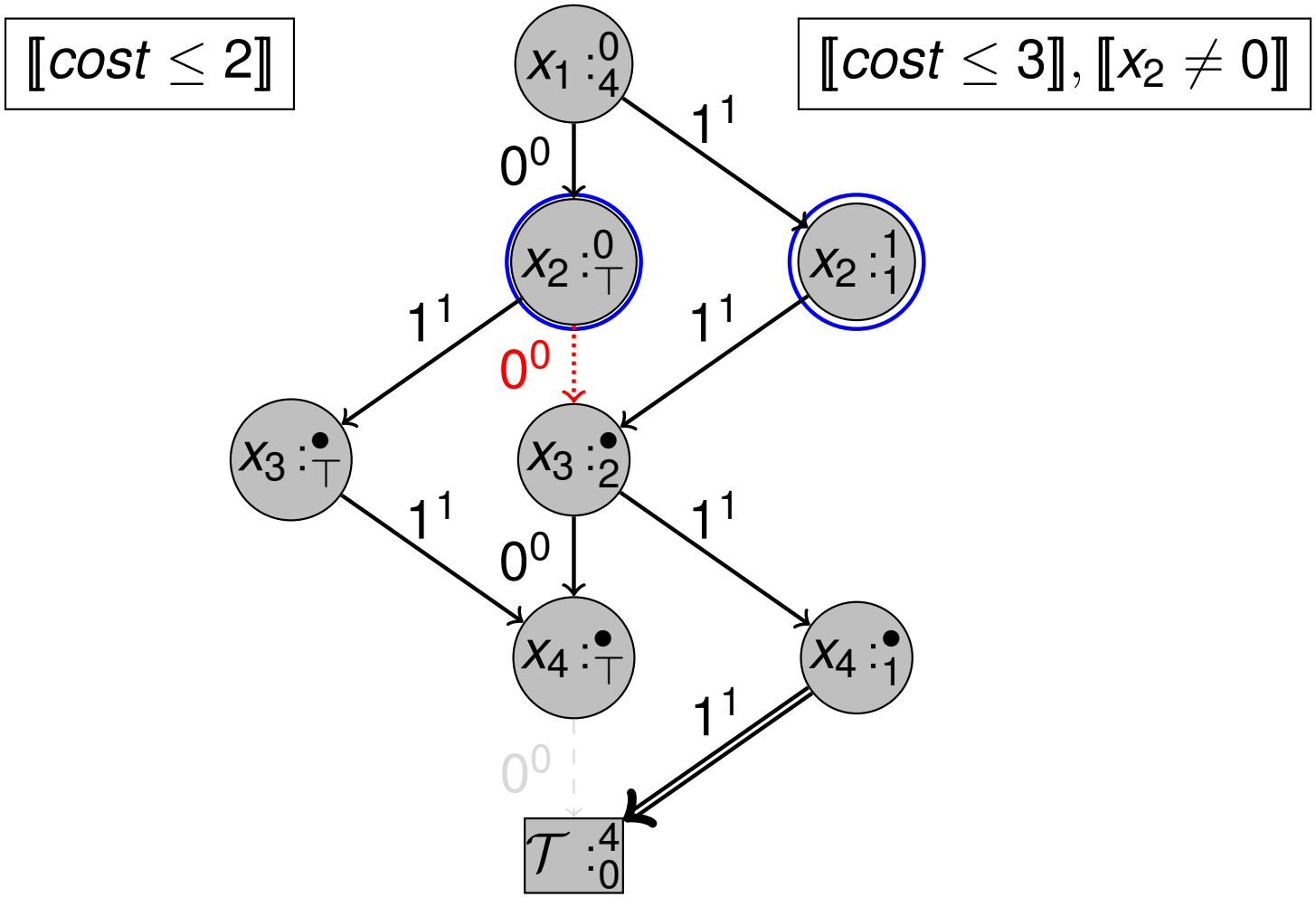

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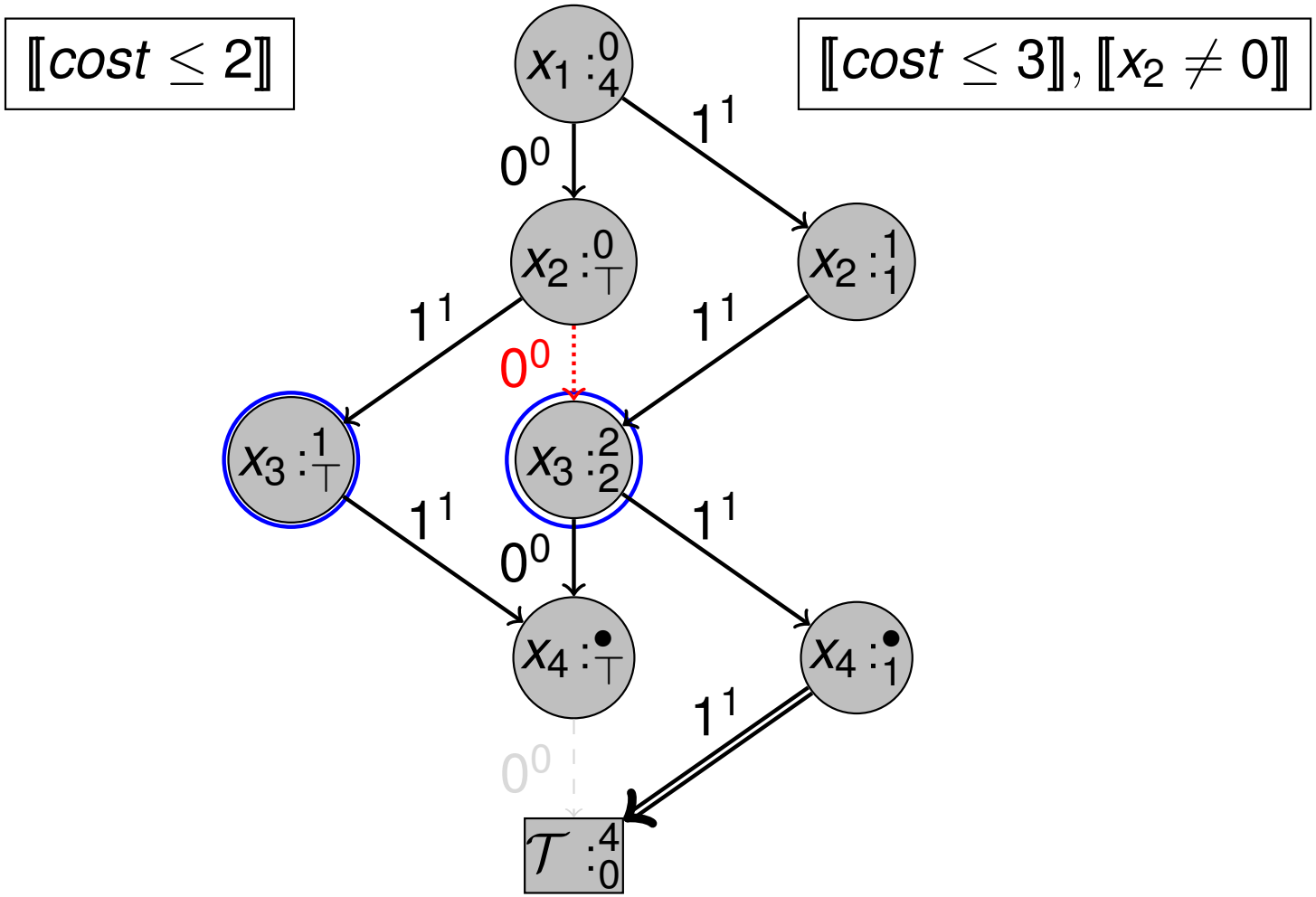

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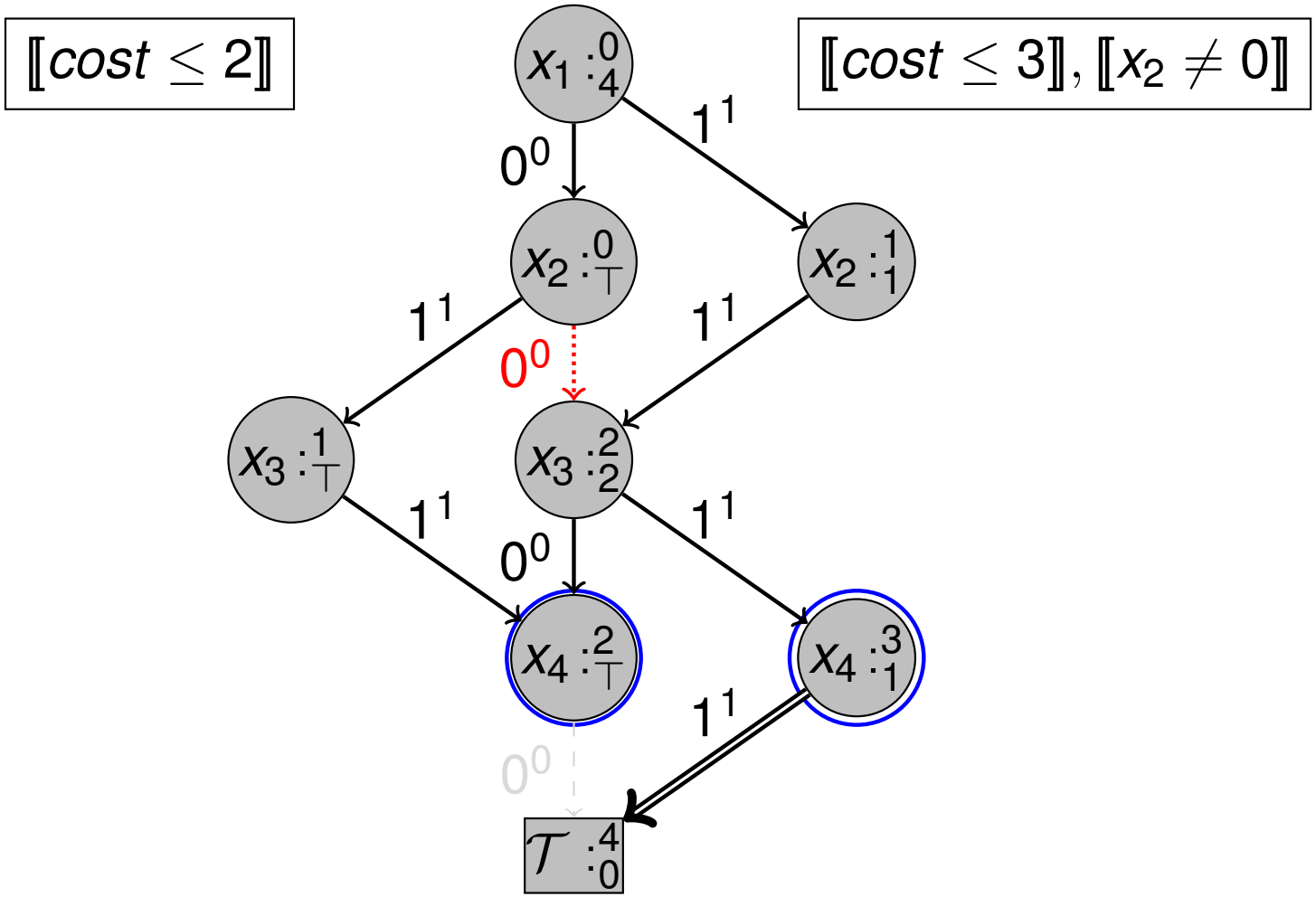

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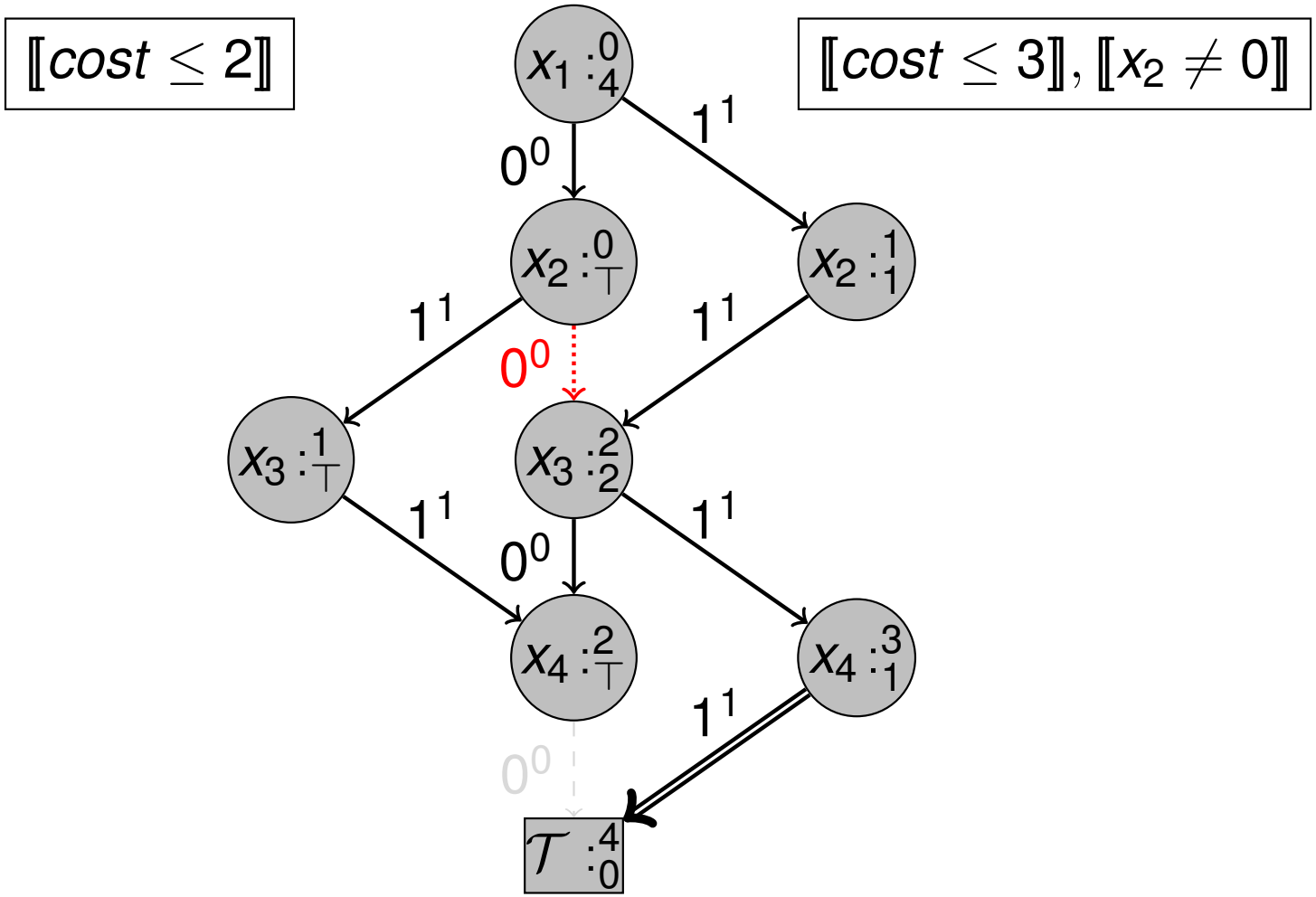

Explanation

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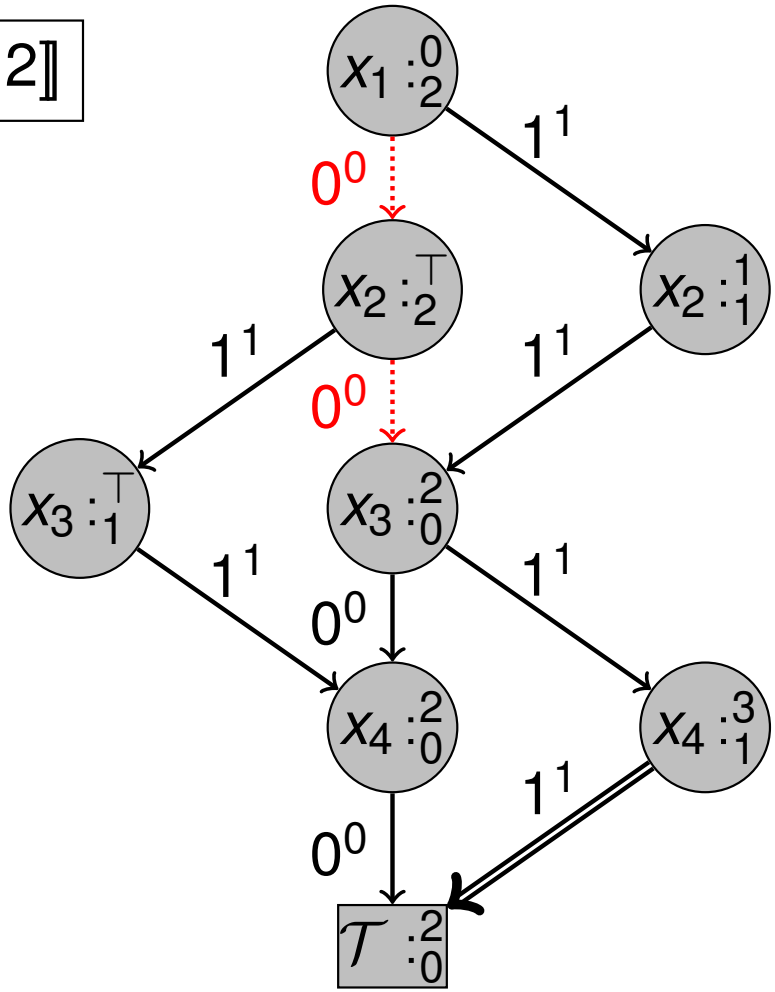
This still requires two full traversals of the graph in the worst case.
Can we do something cheaper?

Greedy Explanation

1. For each edge to be explained
   - compute the smallest in- and out- costs that guarantee infeasibility.
   - If there is slack, allocate it arbitrarily.
2. Sweep the frontier out from the current set of edges, collecting any literals necessary to ensure the path cost.

Example: Explaining $[\![x_4 \neq 1]\!]$ again

$[\![cost \leq 2]\!]$



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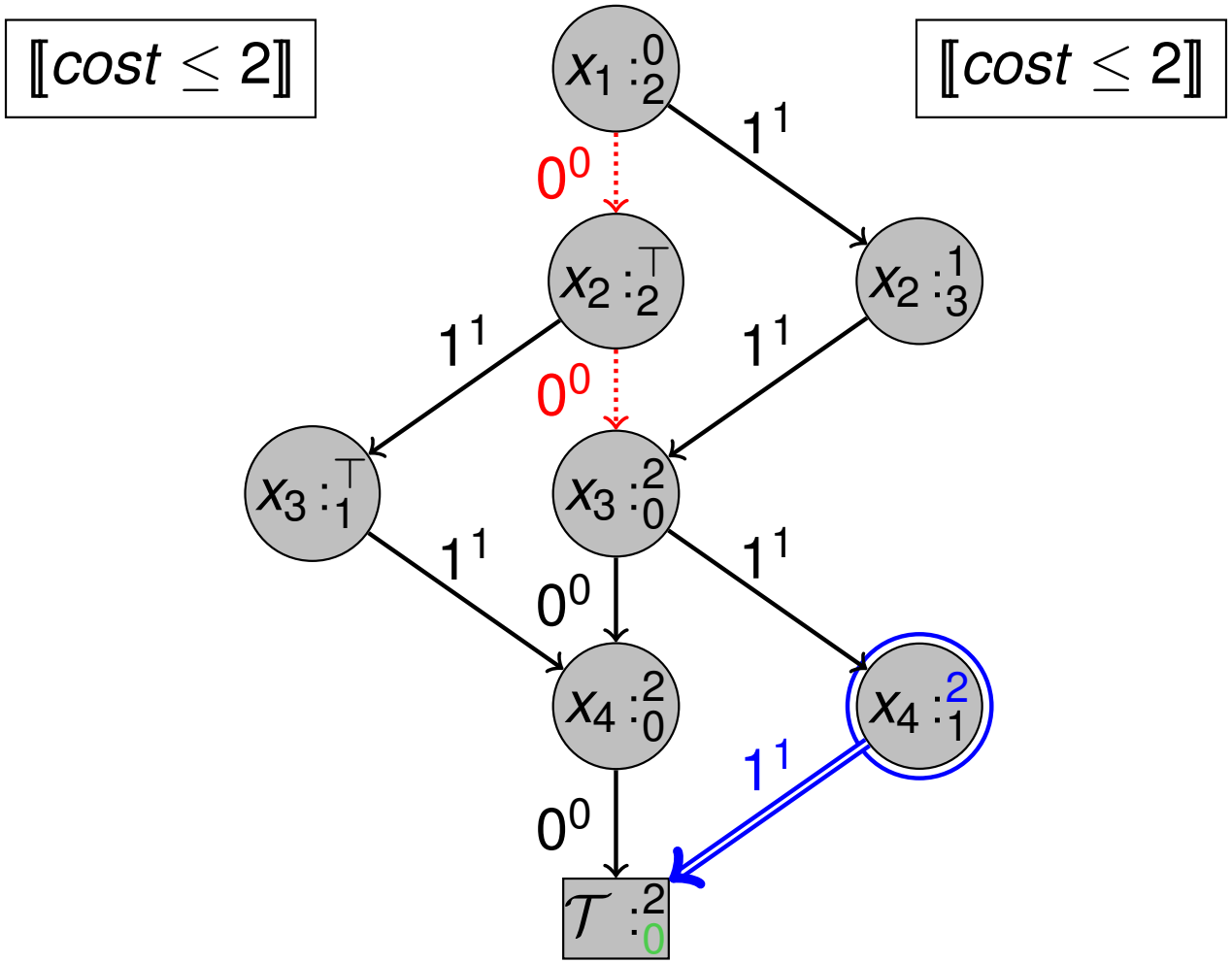

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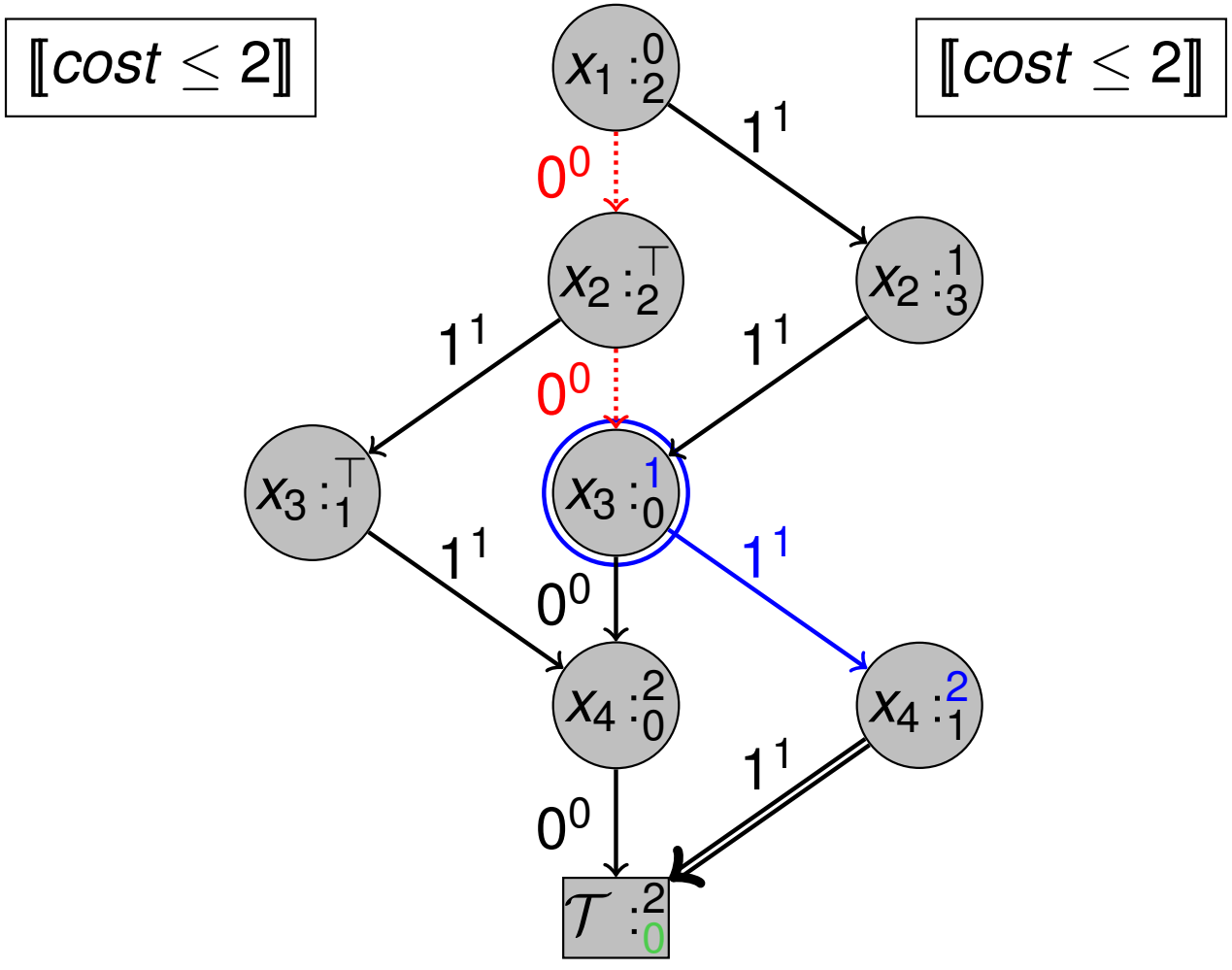

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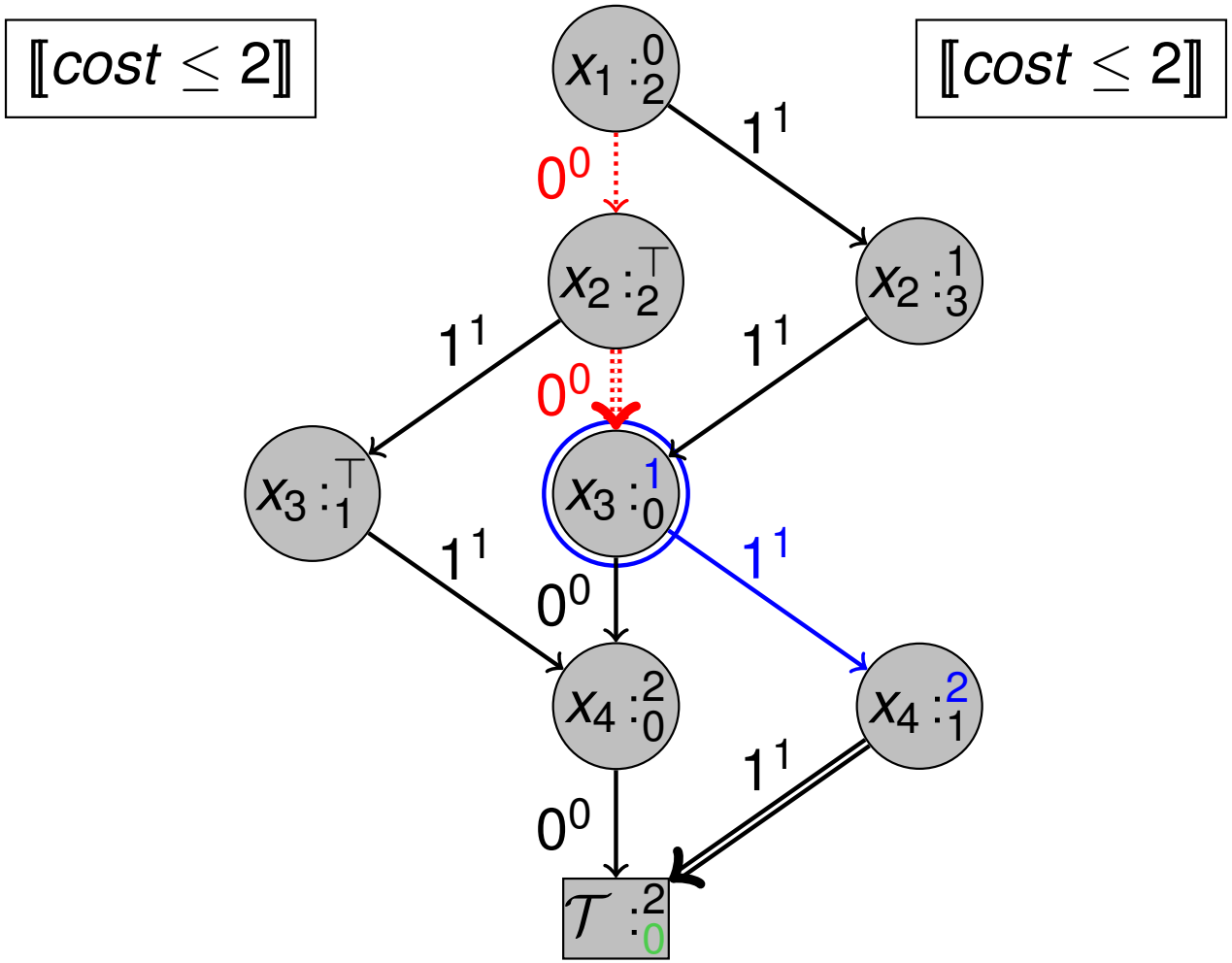

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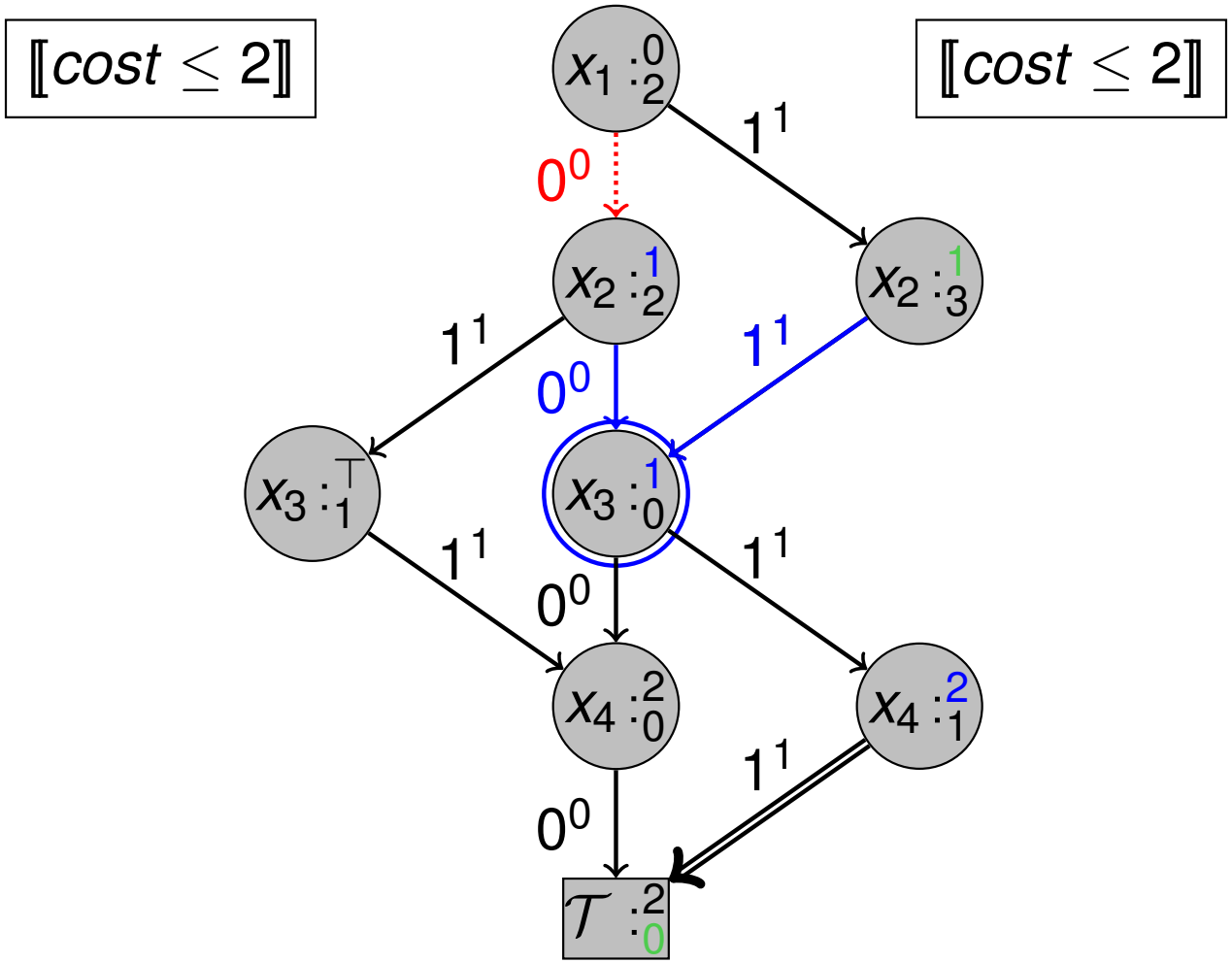

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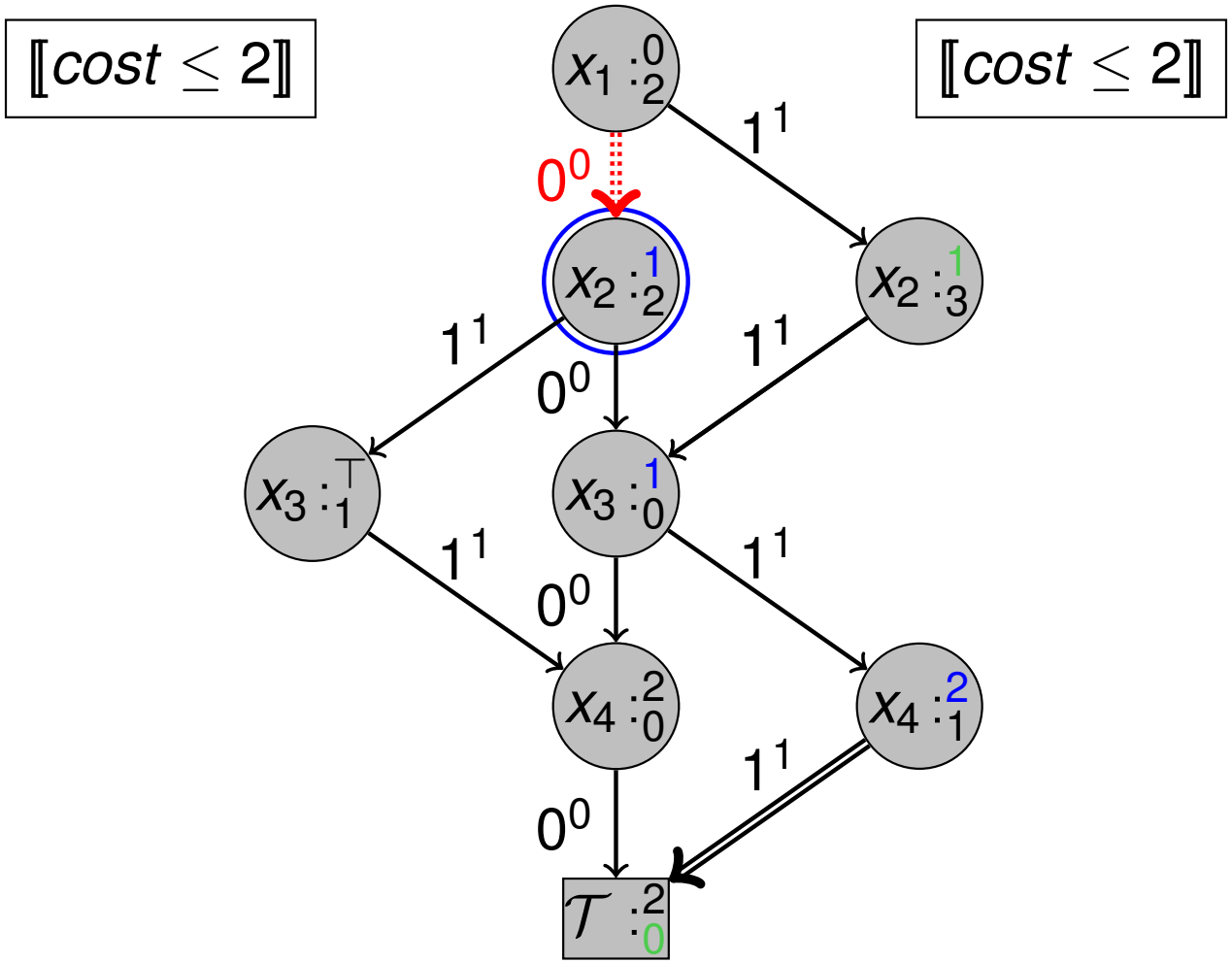

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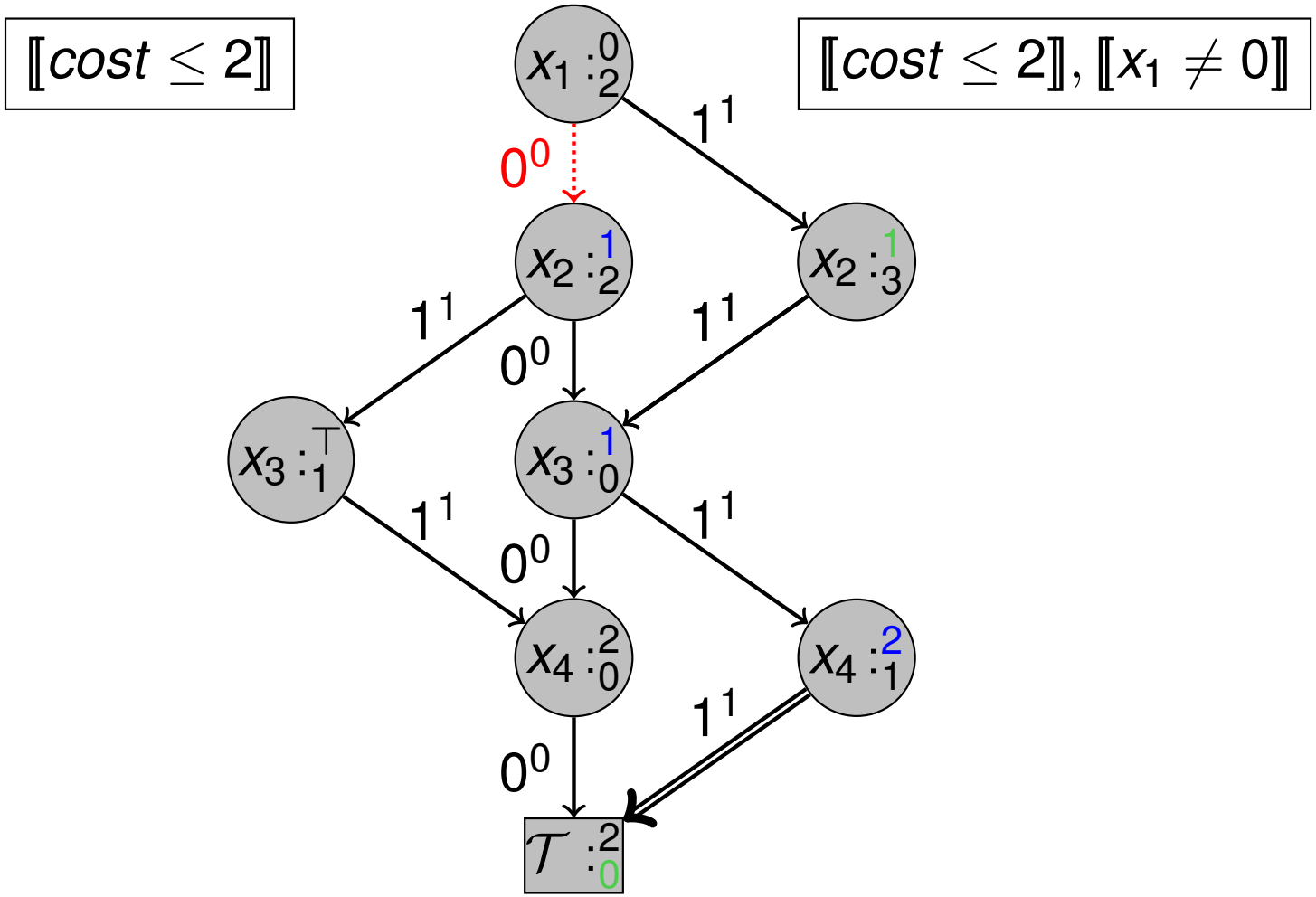

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- To handle infeasible paths, the variable domains must be either:
  - Augmented with ∞
    Not currently supported by solvers
  - Very large (using a big-M encoding)
    Causes solver performance to degrade horrifically
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In practice, this performs worse than the separate decompositions.

Shift Scheduling

Given n employees and k activities, find the minimum cost schedule, subject to:

- At least c_{ij} staff performing activity i at time j.
- A full-time employee works between 6 and 8 hours
  - 1 hour for lunch
  - One 15 minute break both before and after lunch
- A part-time employee works between 3 and 5.75 hours
  - One 15 minute break
- An employee must work on an activity for at least 1 hour, and cannot switch without a break

Shift constraints are described with a context-free grammar, which is then converted into an MDD.
Coverage constraints are implemented with a BDD decomposition.

Experimental Results - Shift Scheduling

Table : Comparison of different methods on shift scheduling problems.

Inst.	dec$_{mdd}$		mdd		ev-mdd		ev-mdd$_l$	
	time	fails	time	fails	time	fails	time	fails
1,2,4	14.51	39700	3.49	21888	0.20	607	**0.17**	635
1,3,6	11.25	40675	19.00	76348	**0.87**	4045	0.91	4156
1,4,6	2.62	7582	0.69	3518	**0.11**	350	0.27	1077
1,5,5	0.41	1585	0.52	3955	0.07	239	**0.06**	238
1,6,6	0.40	1412	0.21	1161	**0.08**	249	0.11	413
1,7,8	4.03	13149	2.43	12046	**0.73**	3838	0.83	4279
1,8,3	0.85	5002	0.39	3606	**0.06**	219	0.07	262
1,10,9	17.55	44222	19.77	68688	**1.23**	5046	1.31	7419
2,1,5	0.14	691	0.24	1490	0.02	78	**0.01**	45
2,2,10	—	—	131.29	286747	**43.62**	99583	49.05	100958
2,3,6	188.77	187760	144.99	289568	**2.39**	6443	5.94	13695
2,4,11	—	—	391.59	918438	**42.38**	111567	92.89	220568
2,5,4	25.85	59635	12.18	50340	0.65	1545	**0.48**	1541
2,6,5	83.78	104911	30.27	80046	**6.18**	12100	7.63	16074
2,8,5	90.28	153331	34.69	110917	**4.99**	15507	10.02	26565
2,9,3	6.10	20472	9.17	42105	0.86	1898	**0.47**	1593
2,10,8	349.88	303227	95.61	168720	**8.85**	26331	17.22	37356
Total	—	—	896.53	2139581	**113.29**	289645	187.44	436874
Mean	—	—	52.74	125857.71	**6.66**	17037.94	11.03	25698.47
Geom.	—	—	7.92	31465.82	**0.86**	2667.65	1.03	3428.35

Results for COST-MDD decomposition are omitted; they were awful.

Conclusions

- Edge valued decision diagrams are a powerful generic constraint: generalizing COST-REGULAR
- Adding explanation significantly improves problem solving
  - Trust us!
- Interesting as opposed to MDD:
  - Minimal explanation are more beneficial than greedy explanations

Questions?

COST-MDD variations

- We have concentrated on COST-MDD$(G, [x_1, \ldots, x_m], \leq, cost)$
  - Requires the cost to be less than a bound.
  - Propagation is domain consistent
  - Most useful in cost minimization problems
- Other variations:
  - COST-MDD$(G, [x_1, \ldots, x_m], \geq, cost)$
    - essentially the same, negate the arc costs and cost variable!
  - COST-MDD$(G, [x_1, \ldots, x_m], =, cost)$
    - Domain propagation is NP-hard
    - In practice treat as the conjunction COST-MDD$(G, [x_1, \ldots, x_m], \leq, cost)$ $\wedge$ COST-MDD$(G, [x_1, \ldots, x_m], \geq, cost)$