The imagination driving Australia's ICT future.

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The SeqBin Constraint Revisited

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UBIA, INRA, and NICTA and University of NSW

Australian Government

Department of Communications, Information Technology and the Arts

Australian Research Council

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Outline

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1. The SeqBin constraint
2. Related work
3. New DC filtering algorithm

SeqBin

The SeqBin constraint
[IJCAI'11,Petit et al.]

Motivation

SeqBin

SeqBin is a meta-constraint useful for
(over-constrained) scheduling and rostering problems

SeqBin

SeqBin is a meta-constraint useful for (over-constrained) scheduling and rostering problems

- Change
- Smooth
- IncreasingNValue

SeqBin

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7$$

SeqBin

$$\text{SeqBin}(N, [X_1, \ldots, X_7], C, B)$$

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7$$

SeqBin

a hard constraint

$$\text{SeqBin}(N, [X_1, \ldots, X_7], C, B)$$

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SeqBin

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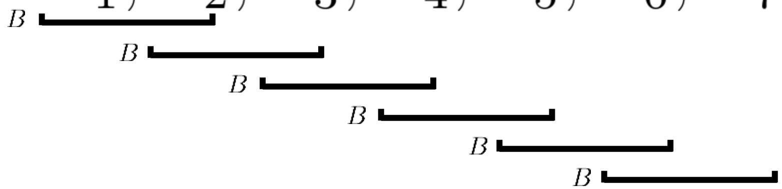


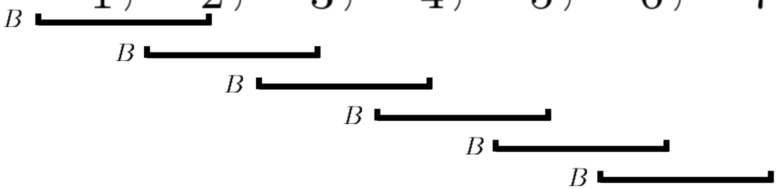
SeqBin

a soft constraint → C

a hard constraint → B

$$\text{SeqBin}(N, [X_1, \ldots, X_7], C, B)$$

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7$$



SeqBin

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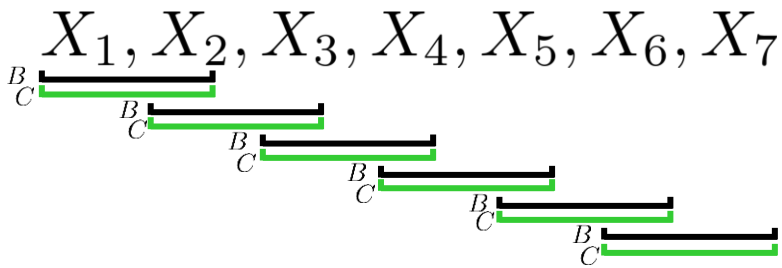
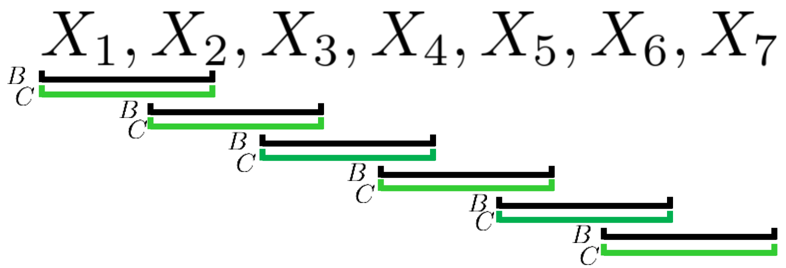


SeqBin

the nb of violated C — a soft constraint — a hard constraint

$$\text{SeqBin}(N, [X_1, \ldots, X_7], C, B)$$

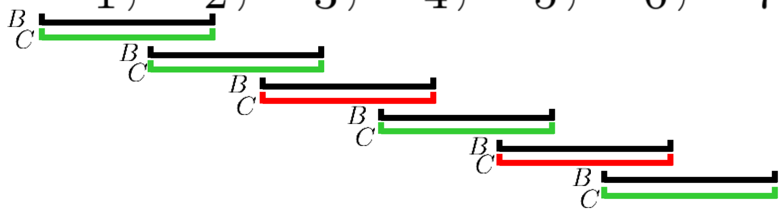


SeqBin

the nb of violated C — a soft constraint — a hard constraint

$$\text{SeqBin}(N, [X_1, \ldots, X_7], C, B)$$

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7$$



SeqBin

the nb of violated C → N; a soft constraint → C; a hard constraint → B

$$\text{SeqBin}(N, [X_1, \ldots, X_7], C, B)$$

holds iff

1. $B(X_i, X_{i+1})$ holds along $[X_1, \ldots, X_n]$
2. $c(X) + 1 = N$, where $c(X)$ is the nb of violated $C(X_i, X_{i+1})$ along $[X_1, \ldots, X_n]$.

SeqBin

$$\text{SeqBin}(N = 3, [X_1, \ldots, X_7], C = \{(1,1)\}, B = \text{True})$$

SeqBin

$$\text{SeqBin}(N = 3, [X_1, \ldots, X_7], C = \{(1,1)\}, B = \text{True})$$

Variables	X_1	X_2	X_3	X_4	X_5	X_6	X_7

SeqBin

$$\text{SeqBin}(N = 3, [X_1, \ldots, X_7], C = \{(1,1)\}, B = \text{True})$$

Variables	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Domains	1	1 0	1	1 0	1	1 0	1

SeqBin

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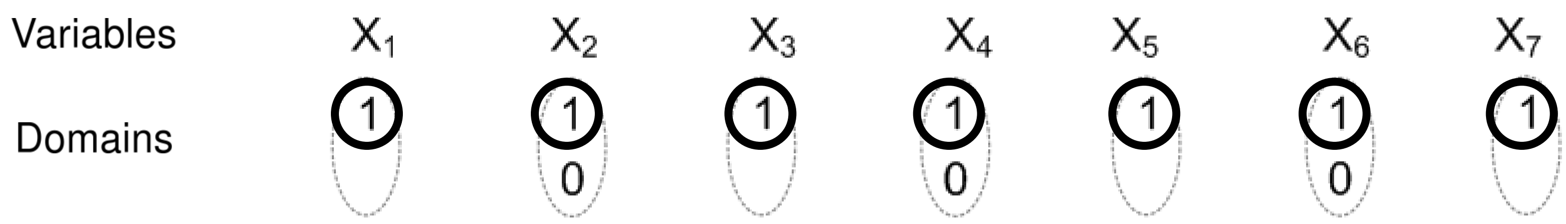

SeqBin

$$\text{SeqBin}(N = 3, [X_1, \ldots, X_7], C = \{(1,1)\}, B \overset{\checkmark}{=} \text{True})$$

Variables	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Domains	1	1	1	1	1	1	1
		0		0		0	

SeqBin

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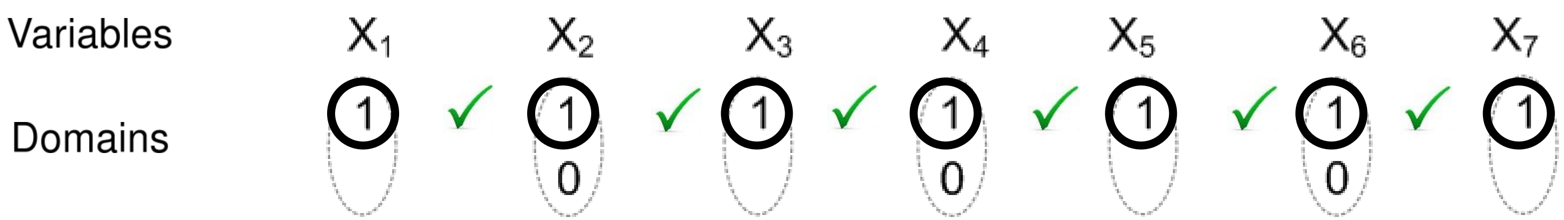


SeqBin

$$\text{SeqBin}(N = 3, [X_1, \ldots, X_7], C = \{(1,1)\}, B \overset{\checkmark}{=} \text{True})$$

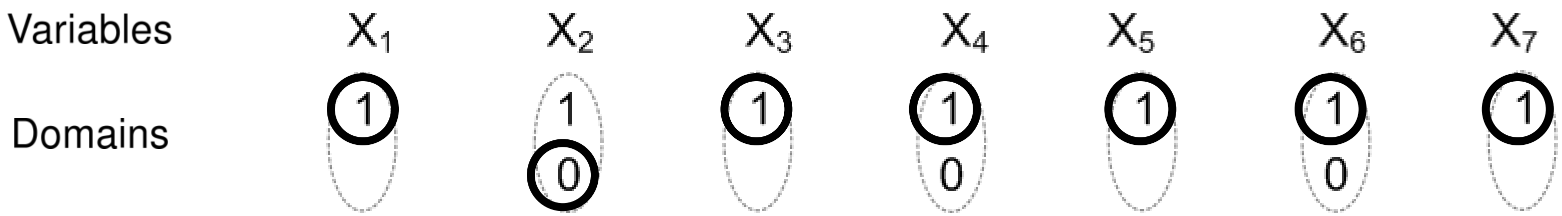
Variables	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Domains	(1)	(1), 0	(1)	(1), 0	(1)	(1), 0	(1)

✓ between each pair of consecutive variables.

c(X) = 0

SeqBin

$$\text{SeqBin}(N = 3, [X_1, \ldots, X_7], C = \{(1,1)\}, B = \text{True})$$



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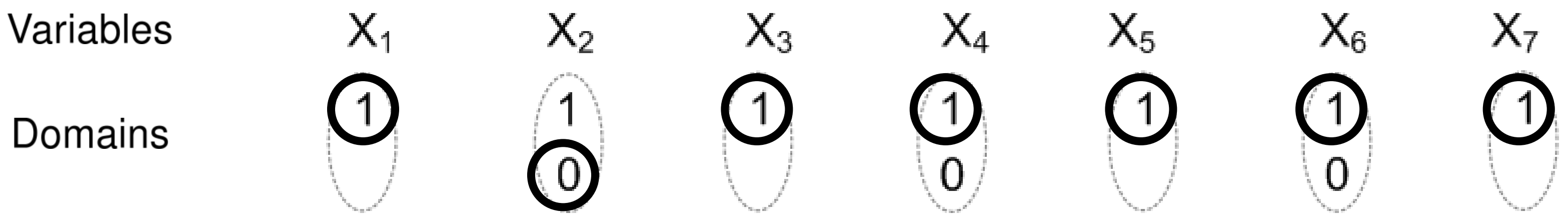

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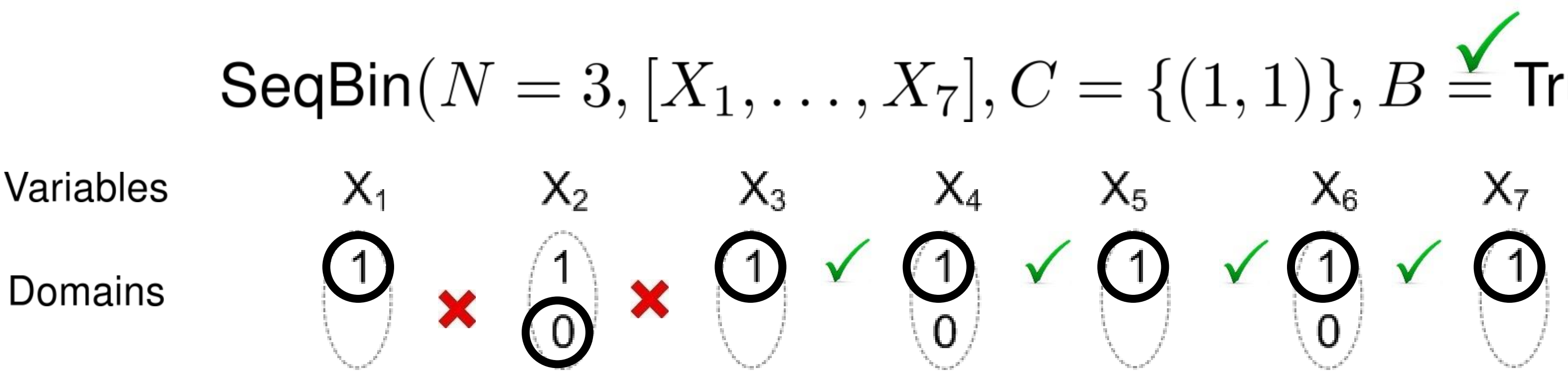

SeqBin

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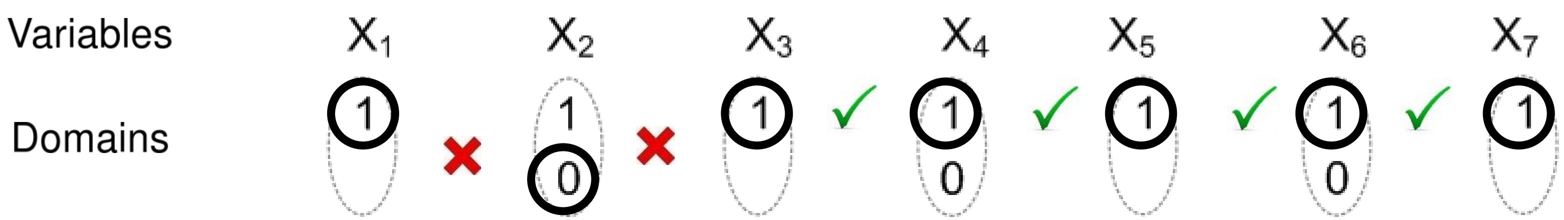
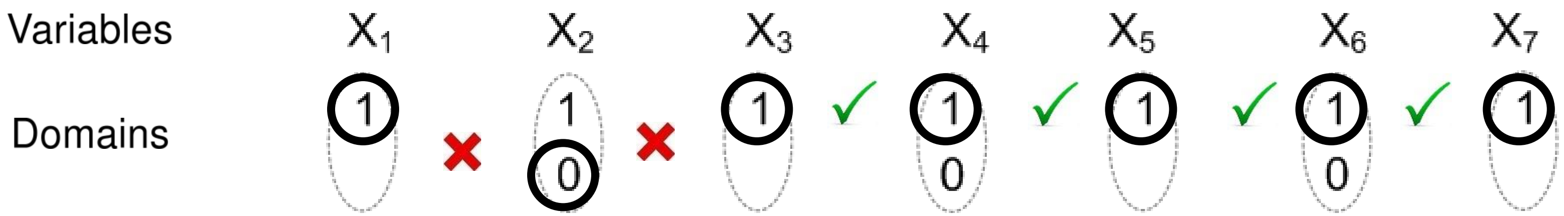


c(X) = 2

SeqBin

$$\text{SeqBin}(N = 3, [X_1, \ldots, X_7], C = \{(1,1)\}, B \stackrel{\checkmark}{=} \text{True})$$



c(X) + 1 = 2 + 1 = N

SeqBin

Disentailment detection

SeqBin

Is there a solution of SeqBin?

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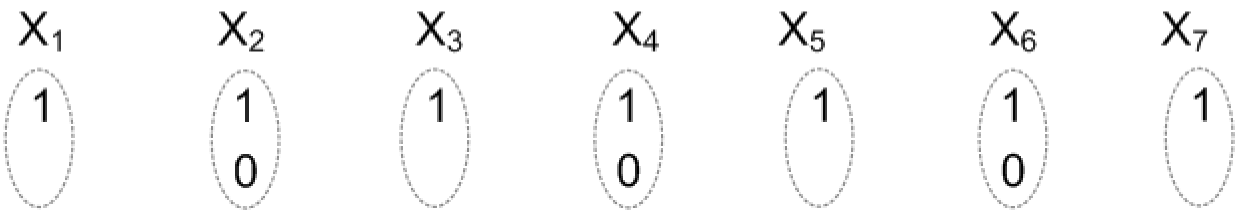
SeqBin (graph representation)

The SeqBin constraint
(equivalent representation)

SeqBin (graph representation)

$$\text{SeqBin}(N, [X_1, \ldots, X_7], C = \{(1,1)\}, B = \text{True})$$



SeqBin (graph representation)

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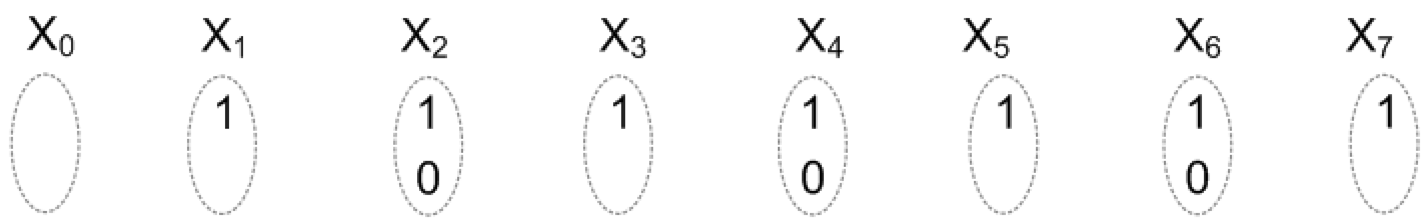

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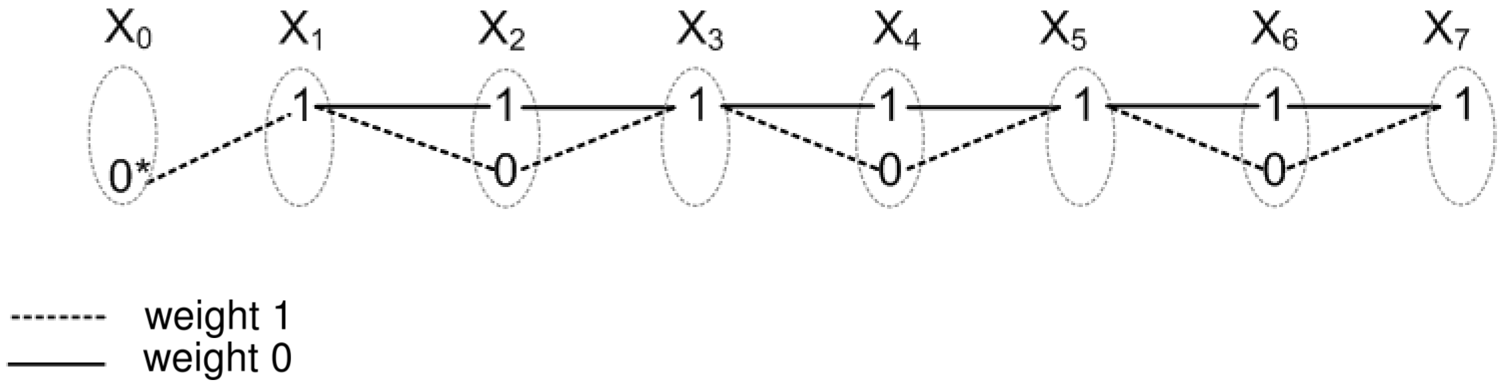

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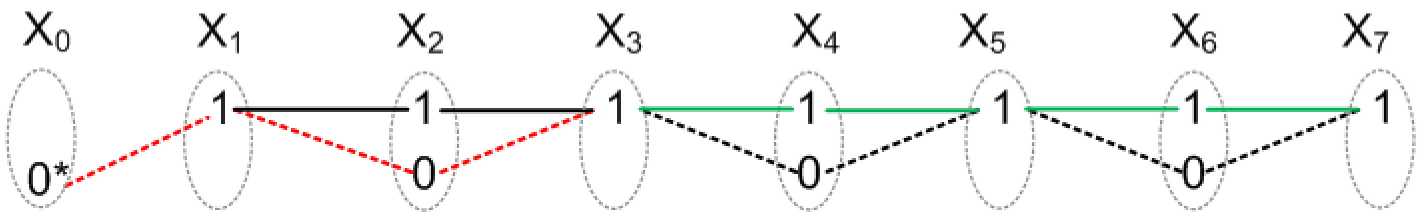

-------- weight 1
——— weight 0

cost of the path is 3

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SeqBin (graph representation)

There exists a bijection between assignments X
of cost s that satisfy B and paths in the graph $G(V, E)$ of cost $s + 1$.

SeqBin

Is there a path in G with cost equal to N?

SeqBin (DP-based propagator)

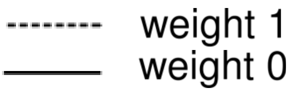
Straightforward approach

Dynamic programming based DC algorithm in $O(n^2 d^2)$

SeqBin (DP-based propagator)

For each vertex in G we compute costs of all possible paths from the sink node.

SeqBin (DP-based propagator)



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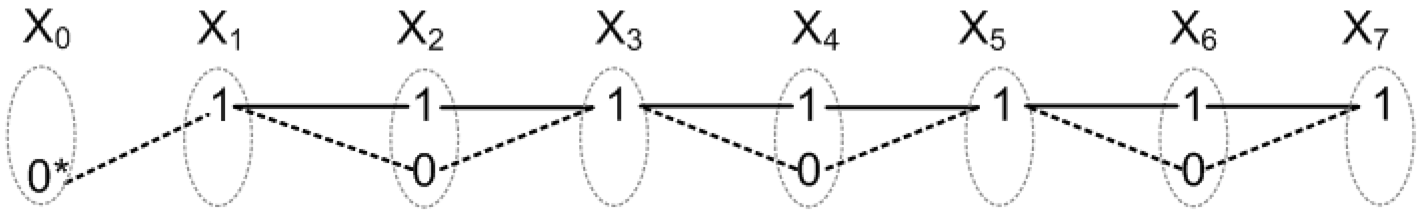
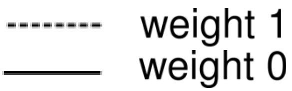


SeqBin (DP-based propagator)



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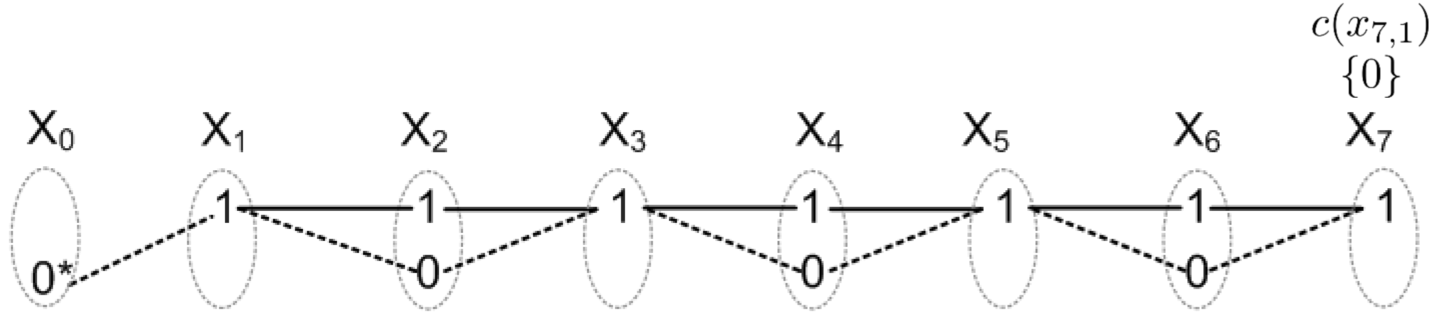

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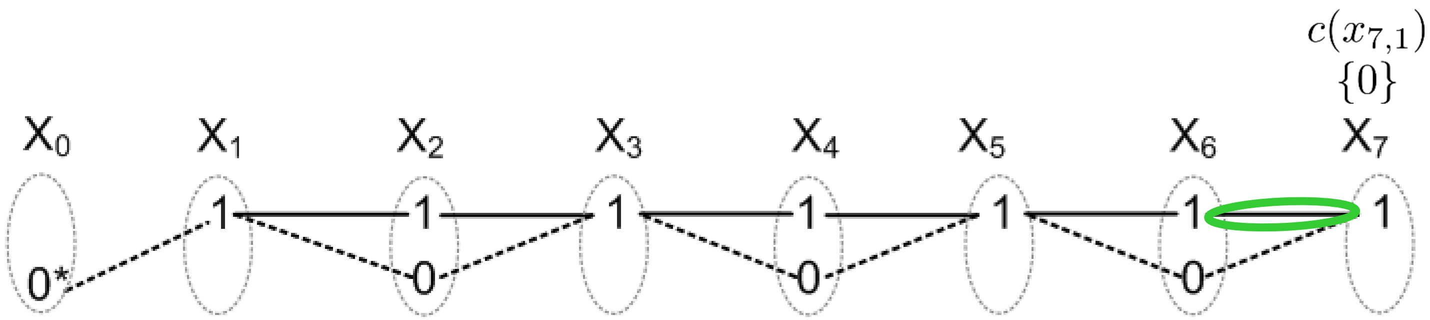

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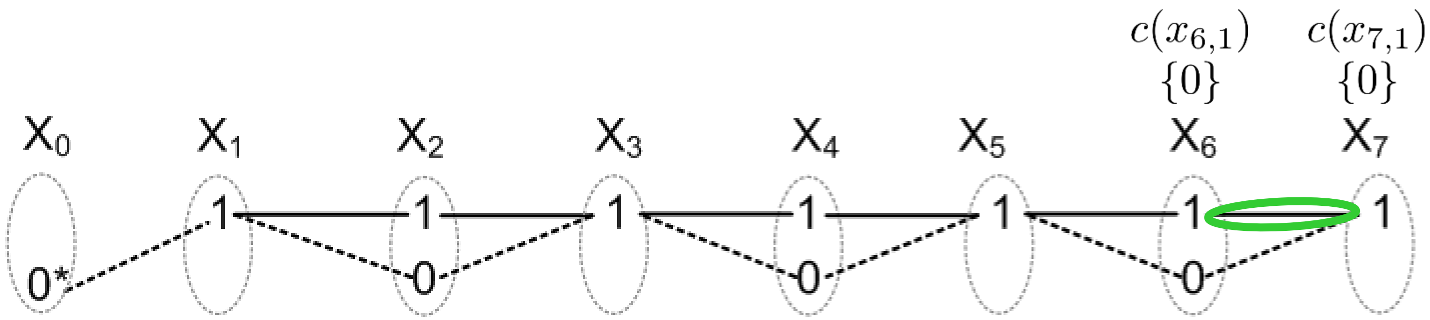

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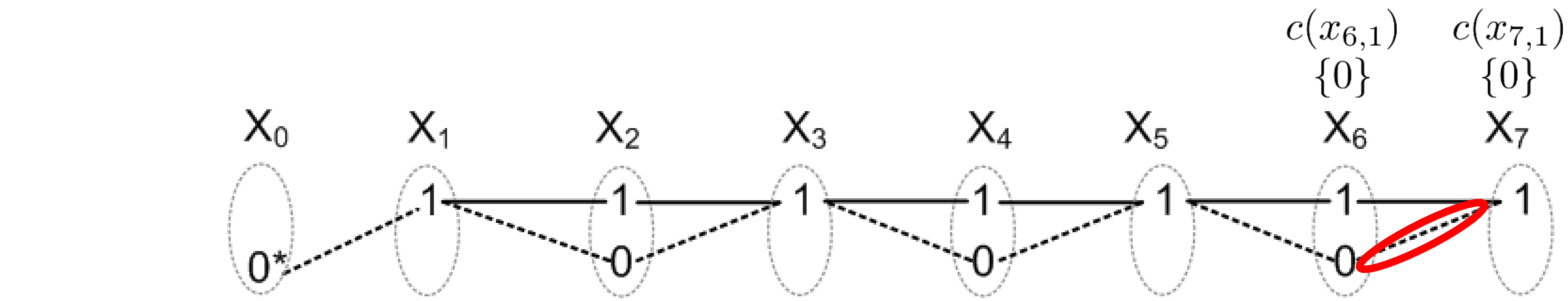

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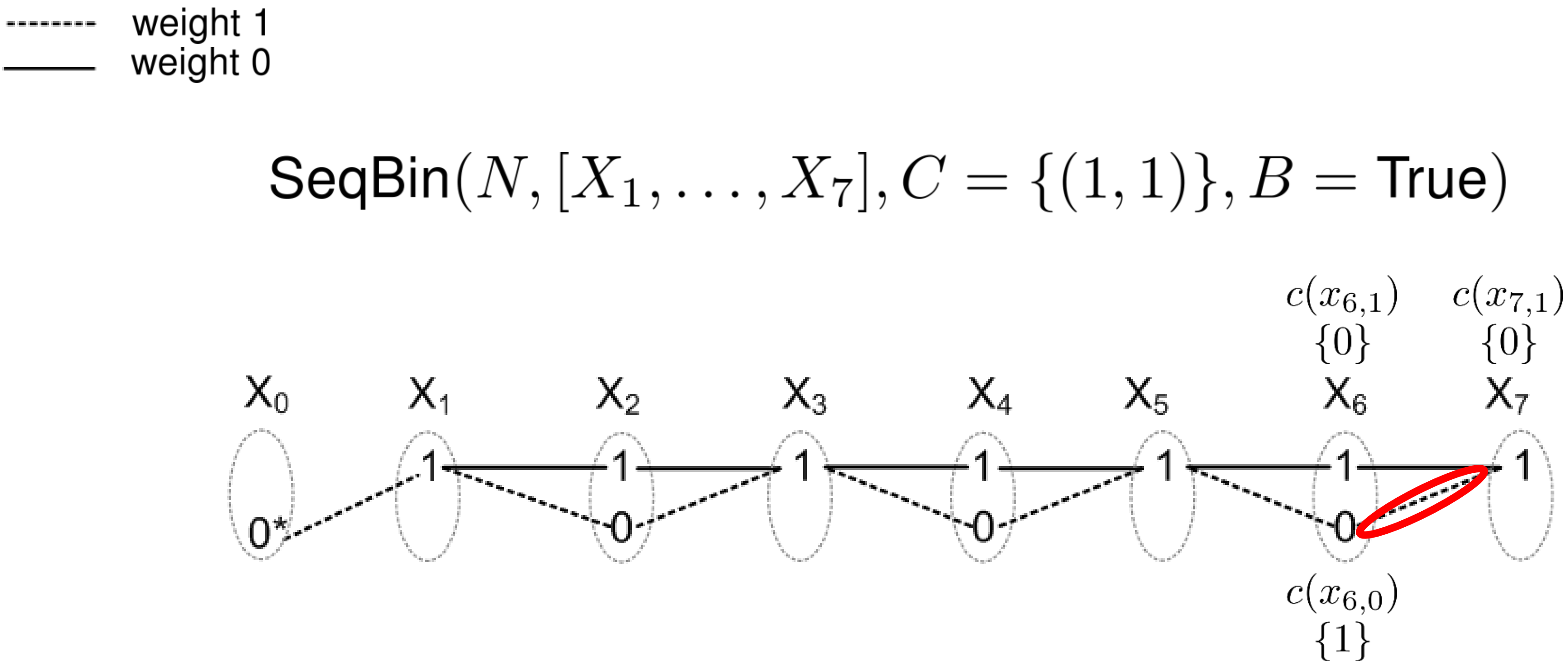
SeqBin (DP-based propagator)



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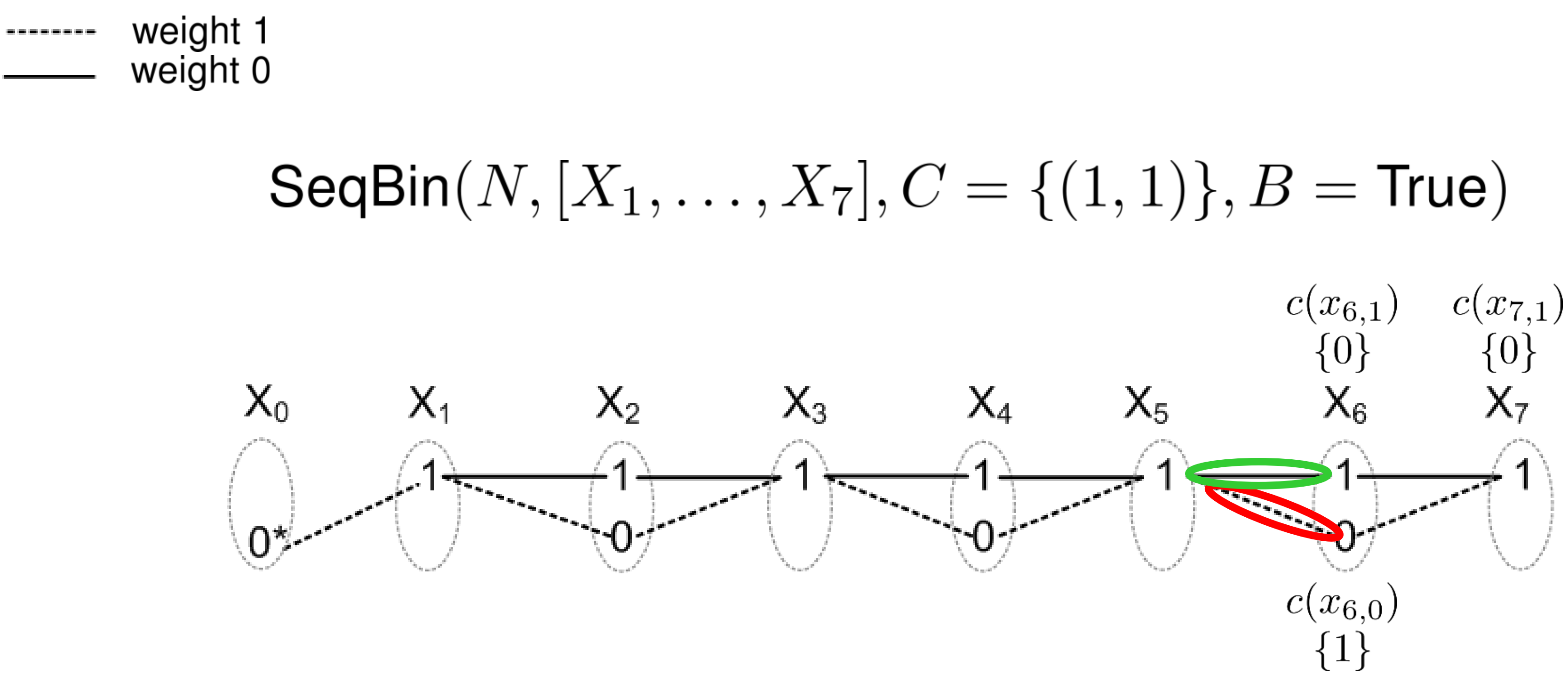
SeqBin (DP-based propagator)

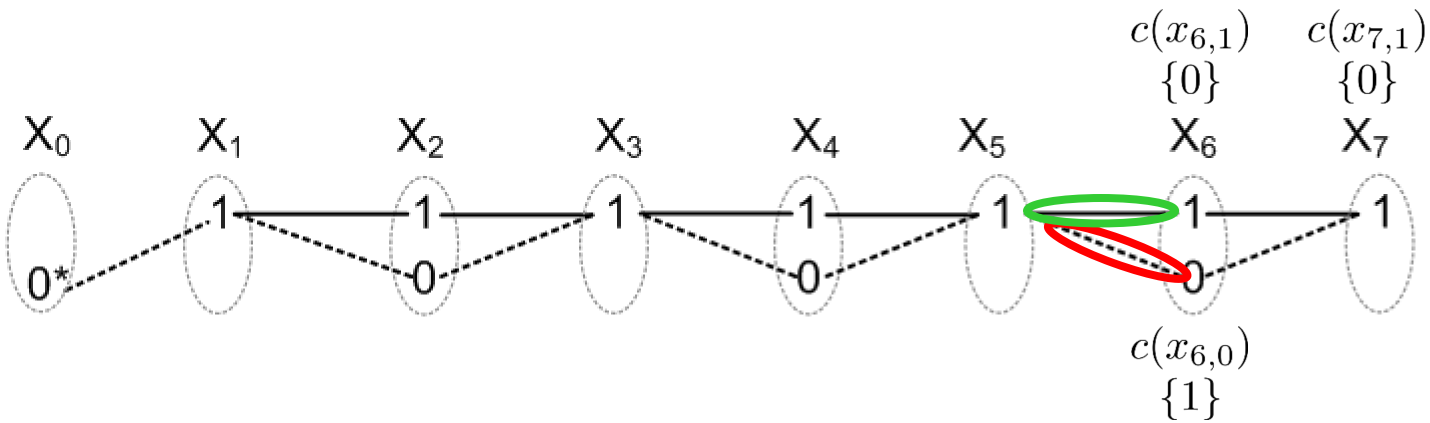


$$\text{SeqBin}(N, [X_1, \ldots, X_7], C = \{(1,1)\}, B = \text{True})$$

SeqBin (DP-based propagator)

- - - - - - - weight 1
——— weight 0

$$\text{SeqBin}(N, [X_1, \ldots, X_7], C = \{(1,1)\}, B = \text{True})$$



$$c(x_{5,1}) = (c(x_{6,1}) \uplus 0) \cup (c(x_{6,0}) \uplus 1)$$

SeqBin (DP-based propagator)

-------- weight 1

——— weight 0

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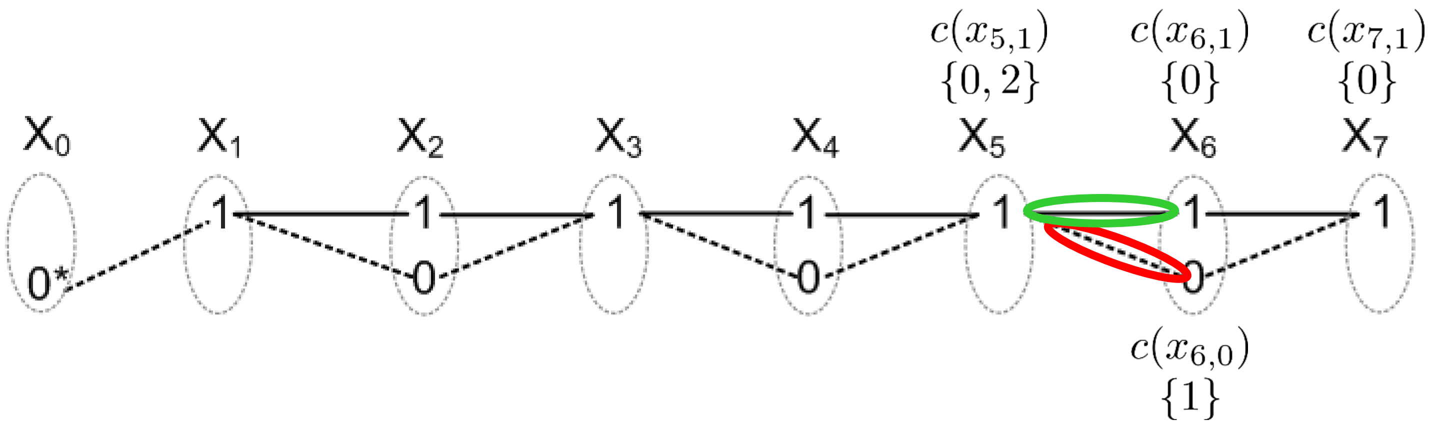

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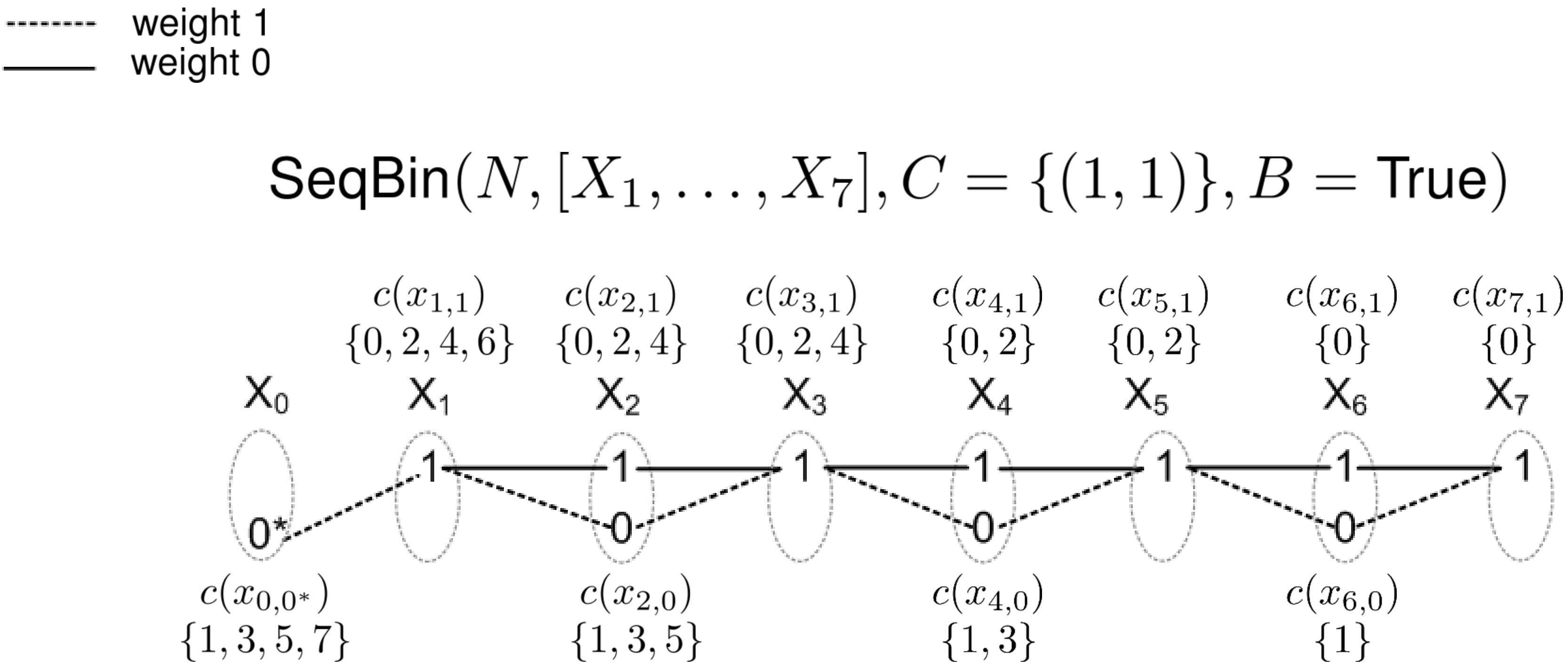

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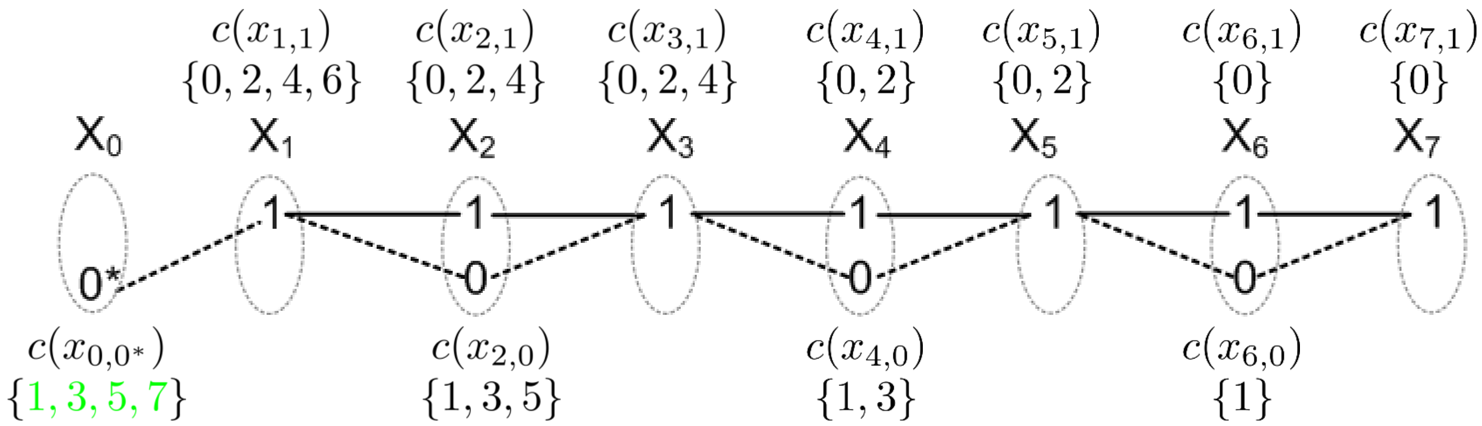
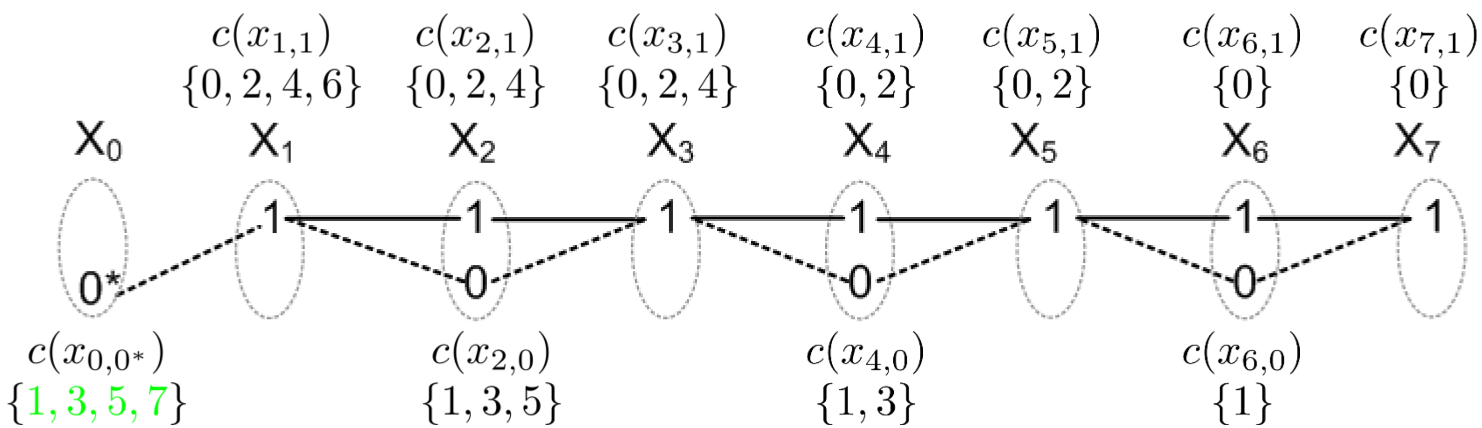


SeqBin (DP-based propagator)

- - - - - - - - weight 1
———— weight 0

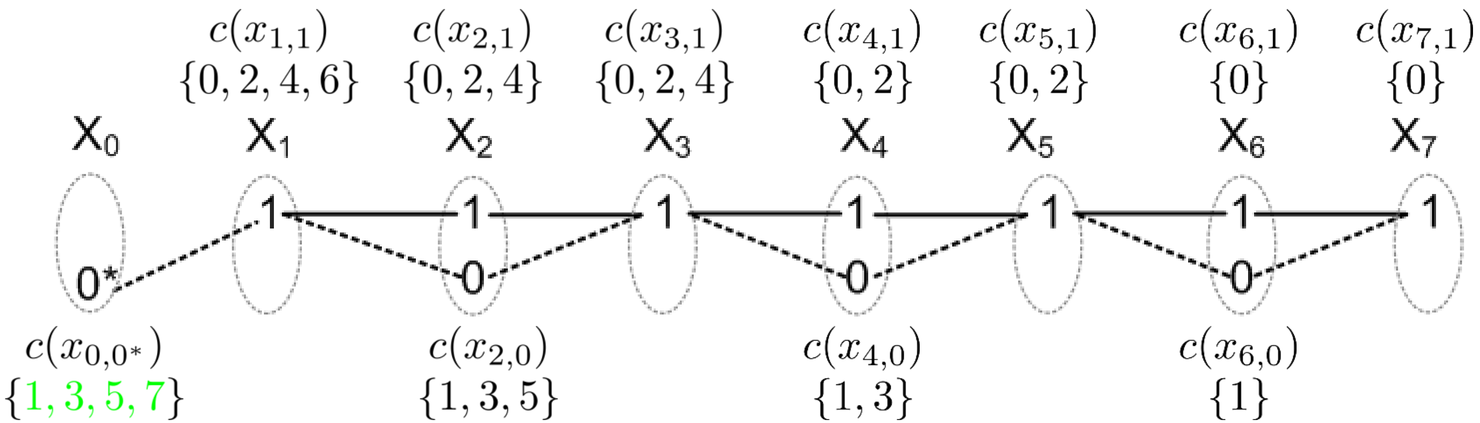
$$\text{SeqBin}(N = 3, [X_1, \ldots, X_7], C = \{(1,1)\}, B = \text{True})$$



SeqBin (DP-based propagator)

-------- weight 1
———— weight 0

$$\text{SeqBin}(N = 4, [X_1, \ldots, X_7], C = \{(1,1)\}, B = \text{True})$$



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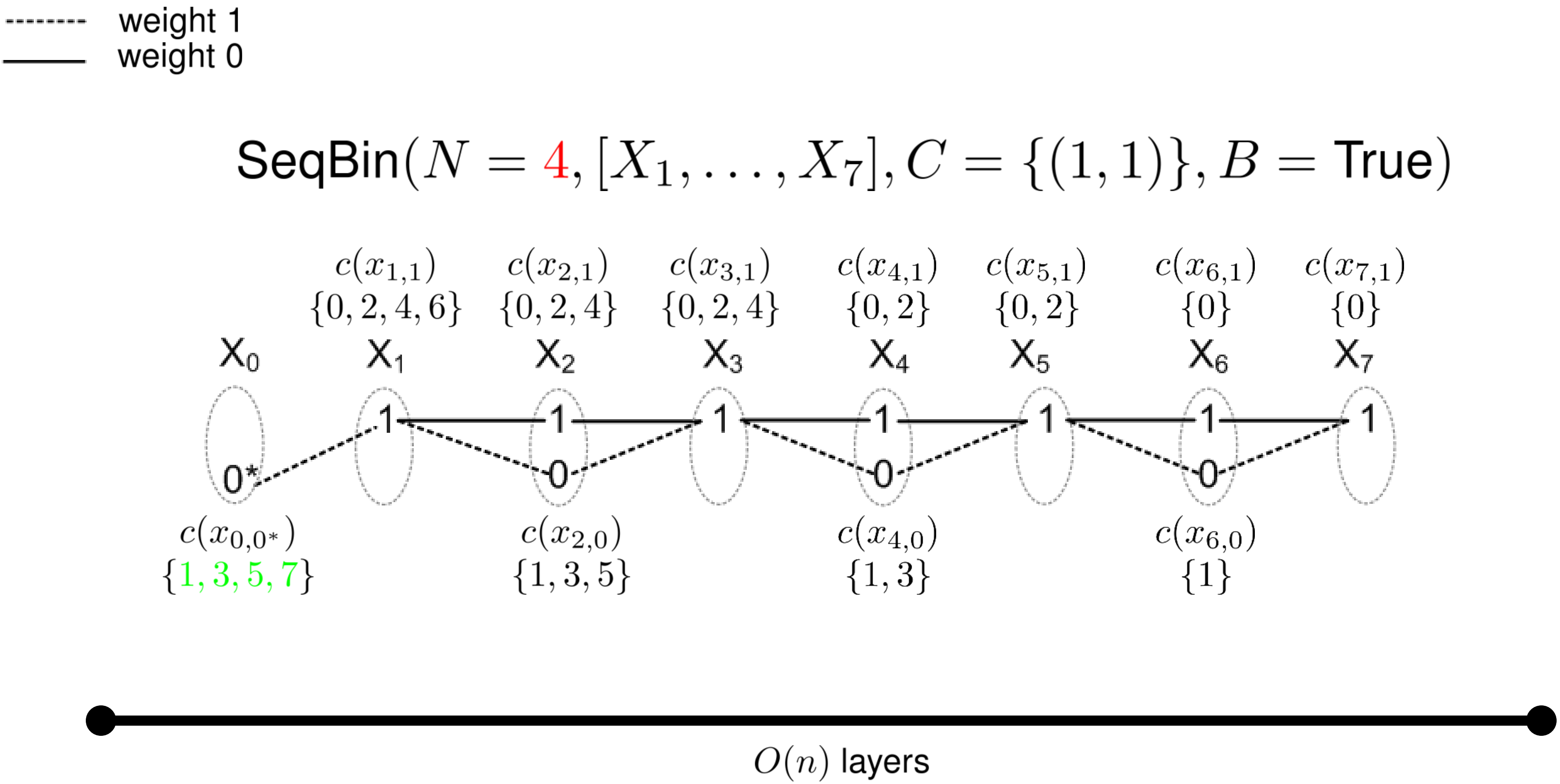
SeqBin (DP-based propagator)



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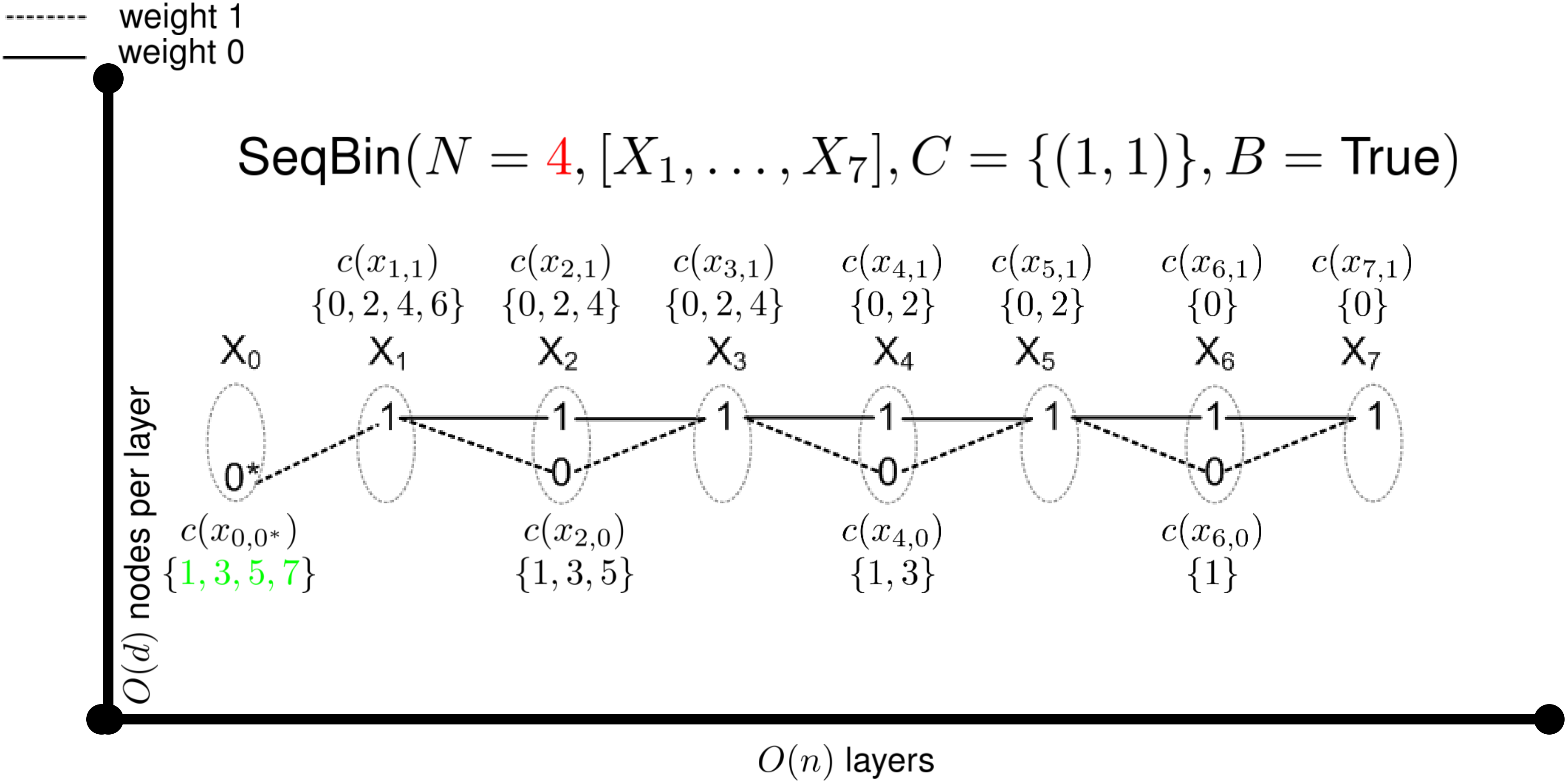

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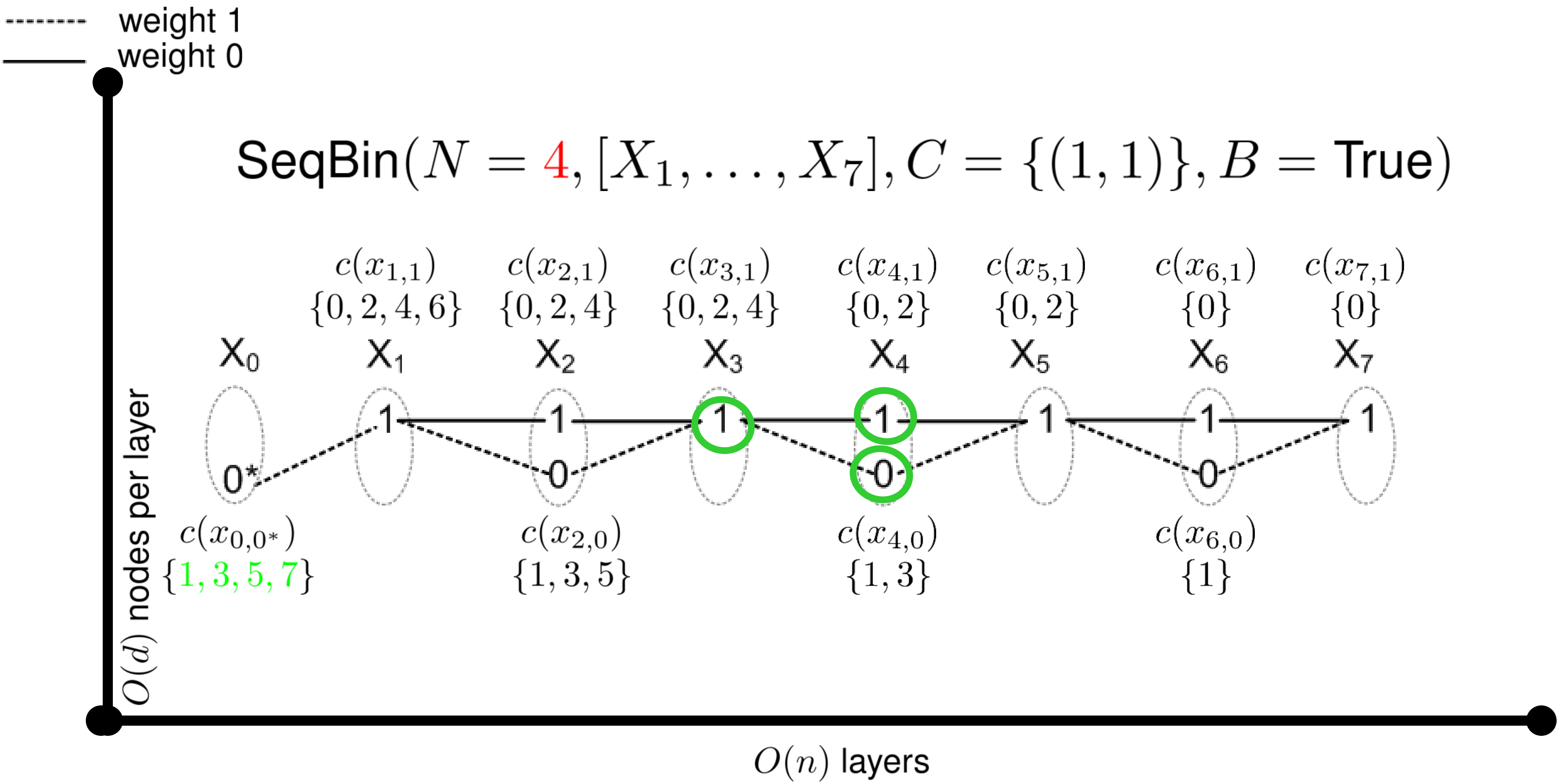
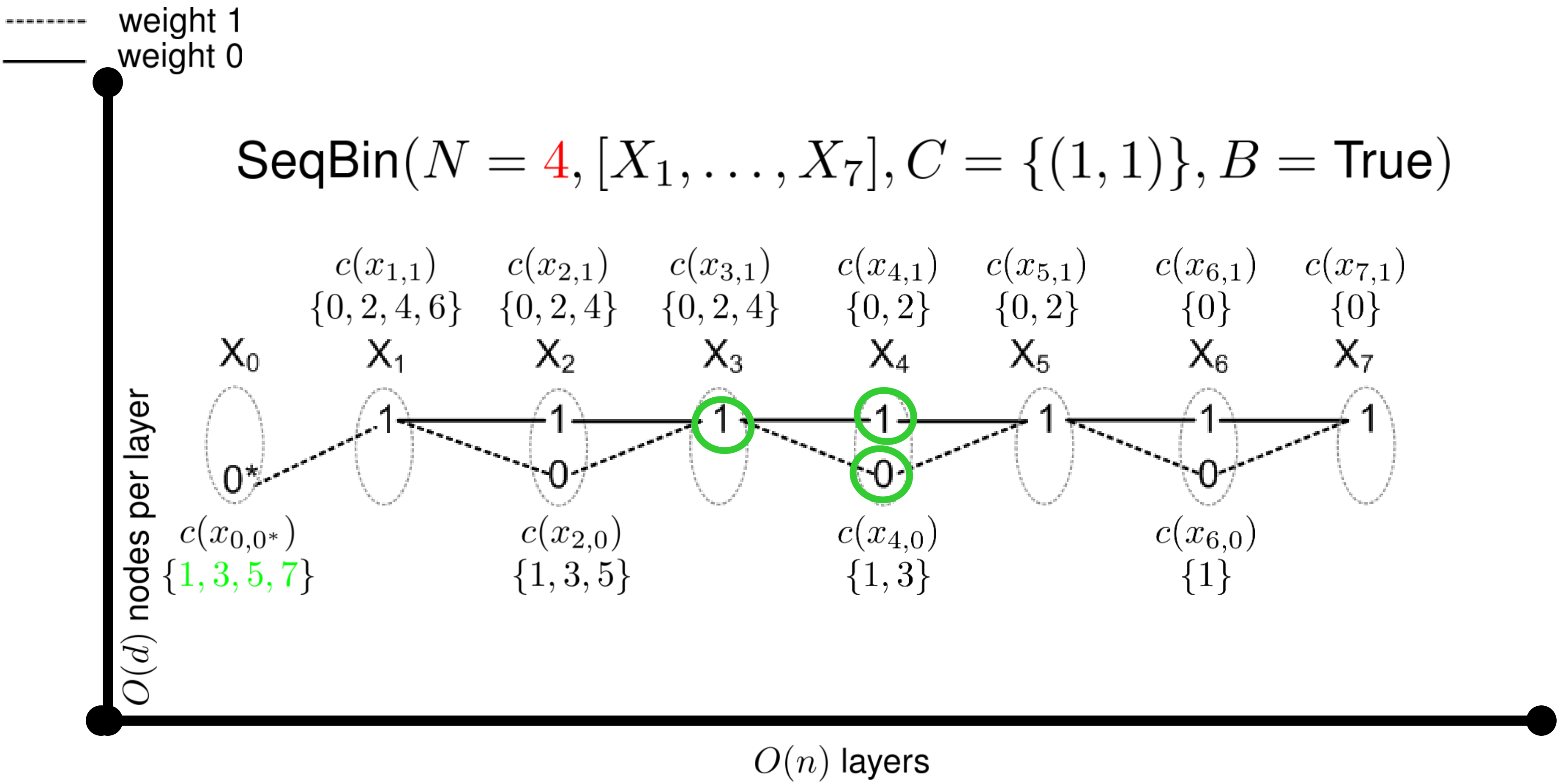


$$c(x_{3,1}) = (c(x_{4,1}) \uplus 0) \cup (c(x_{4,0}) \uplus 1) \qquad O(nd)$$

SeqBin (DP-based propagator)



$$c(x_{i,j}) = \bigcup_{v \in D(X_{i+1})} \left(c(x_{i+1,v}) \uplus w(j,v)\right) \quad O(nd)$$

Related work

Related work
[IJCAI'11,Petit et al.]

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Related work

$$\mathsf{SeqBin}(N, [X_1, \ldots, X_7], C, B)$$

Related work

the monotone constraint

$$\text{SeqBin}(N, [X_1, \ldots, X_7], C, B)$$

Related work

Dynamic programming based filtering algorithm in $O(nd^2)$

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Related work

$$c(x_{i,j}) = \bigcup_{v \in D(X_{i+1})} (c(x_{i+1,v}) \uplus w(j,v)) \quad O(nd)$$

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Related work

– does not detect bounds disentailment
observed independently by Petit et al. , TR'2011

Related work

– does not detect bounds disentailment
observed independently by Petit et al. , TR'2011

– not idempotent (i.e. $f(f(X)) \neq f(X)$)

Related work

Main problem: if B is monotone then

$$c(x_{i,j}) = [l, u]$$

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Related work

Main problem: if B is monotone then

$$c(x_{i,j}) = [l, u]$$

$$c(x_{i,j}) = \bigcup_{v \in D(X_{i+1})} (c(x_{i+1,v}) \uplus w(j, v)) \quad O(d)$$

Related work

Main problem: if B is monotone then

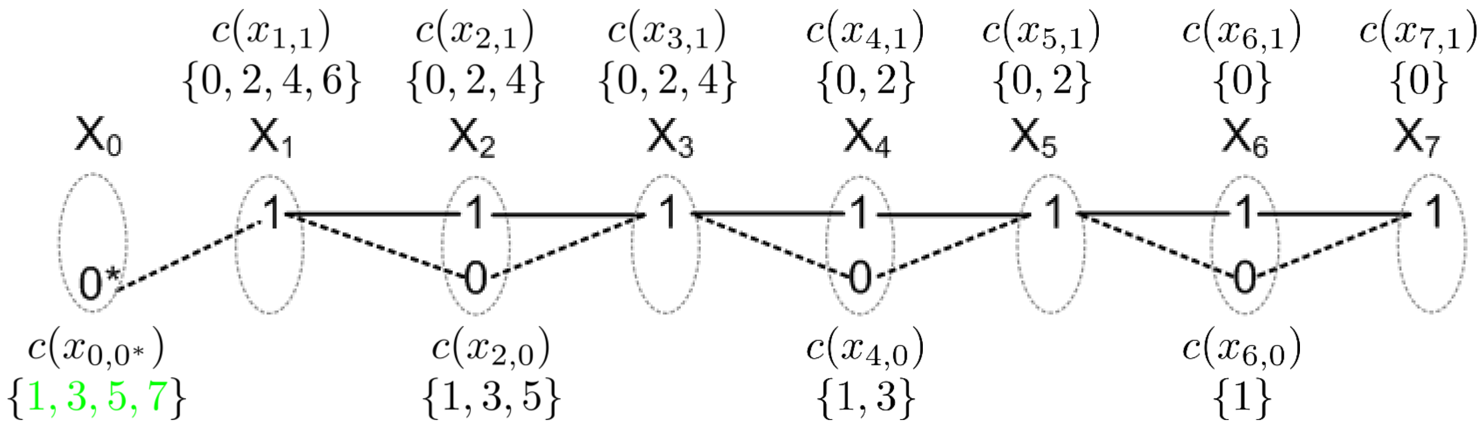
$$\sout{c(x_{i,j}) = [l, u]}$$

$$c(x_{i,j}) = \bigcup_{v \in D(X_{i+1})} \left(c(x_{i+1,v}) \uplus w(j, v)\right) \quad O(d)$$

SeqBin (DP-based propagator)

-------- weight 1
——— weight 0

$$\text{SeqBin}(N, [X_1, \ldots, X_7], C = \{(1,1)\}, B = \text{True})$$



Our contribution:
new DC filtering algorithm in $O(nd^2)$

New DC propagator

Goal: improve DP when B is monotone

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New DC propagator

$$c(x_{i,j}) = \bigcup_{v \in D(X_{i+1})} (c(x_{i+1,v}) \uplus w(j,v)) \quad O(nd)$$

Cost properties

$c(x_{i,j})$ is not an interval

Cost properties

$c(x_{i,j})$ is not even an 'almost' interval (unbounded number of holes!)

Cost properties

$c(x_{i,j})$ has a special structure! 

Uniqueness

$c(x_{i,j})$ is a set of one of the following two forms:

zipper:

Uniqueness

$c(x_{i,j})$ is a set of one of the following two forms:

zipper: $\{2, 4, 6, 8\}$.

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i·zipper:

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$c(x_{i,j})$ is a set of one of the following two forms:

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i·zipper: $[4, 11]$

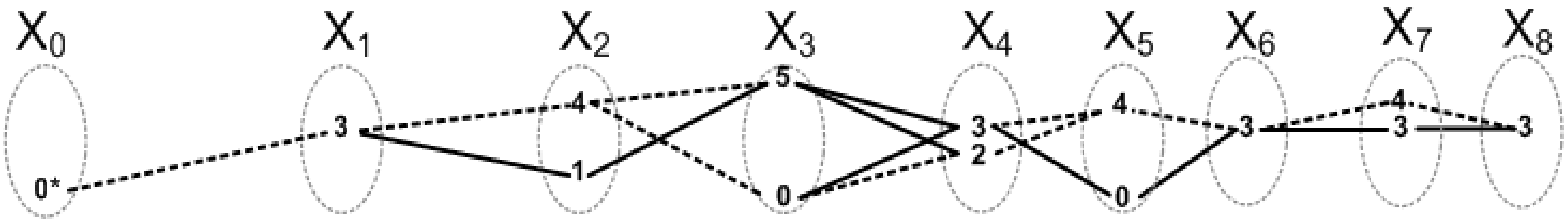
Uniqueness

$c(x_{i,j})$ is a set of one of the following two forms:

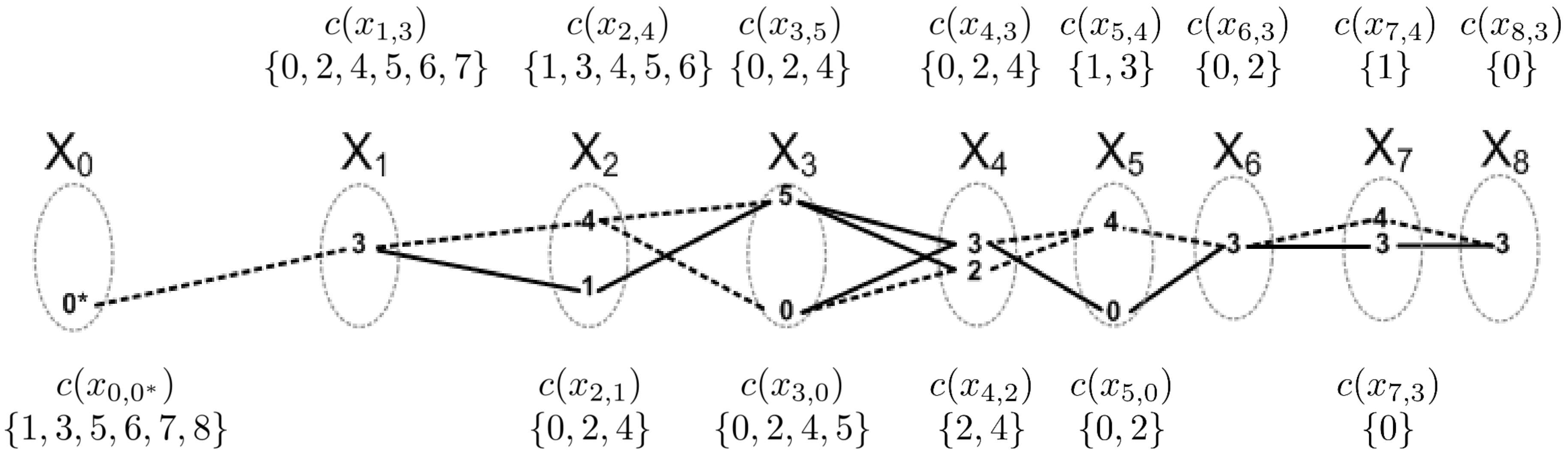
zipper: $\{2, 4, 6, 8\}$.

i·zipper: $\{2, 4\} \cup [4, 11] \cup \{11, 13, 15\}$

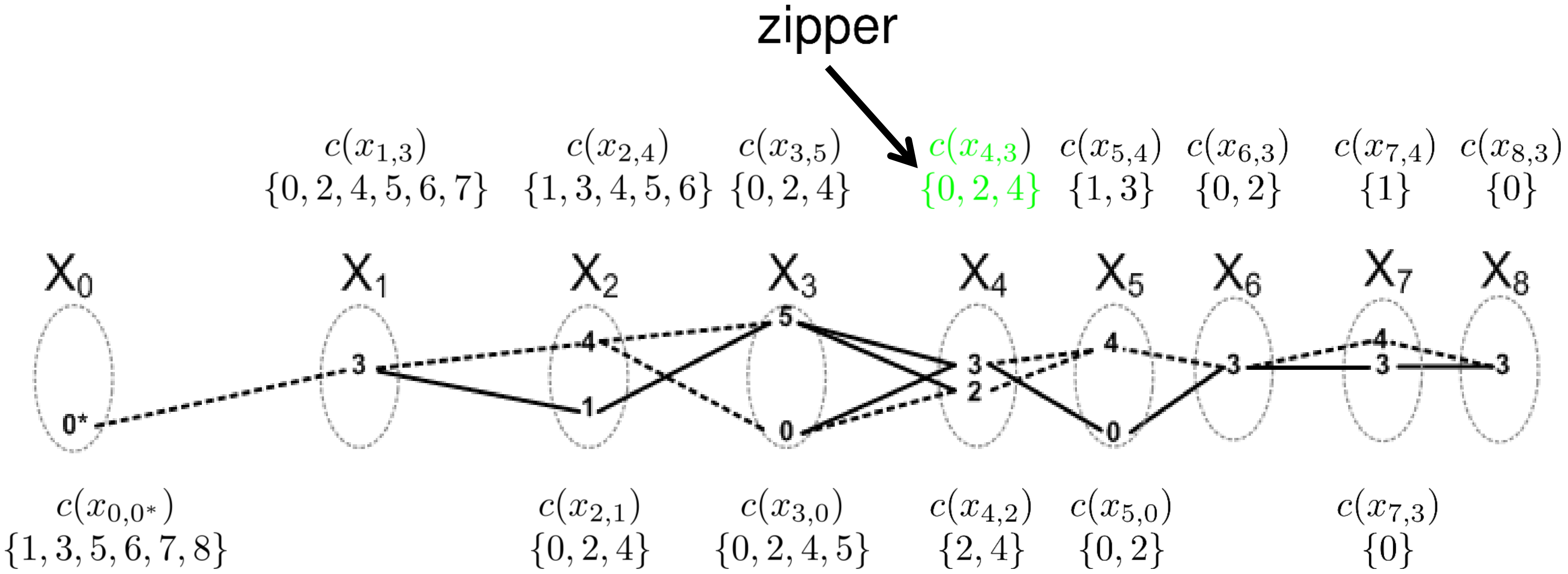
New DC propagator



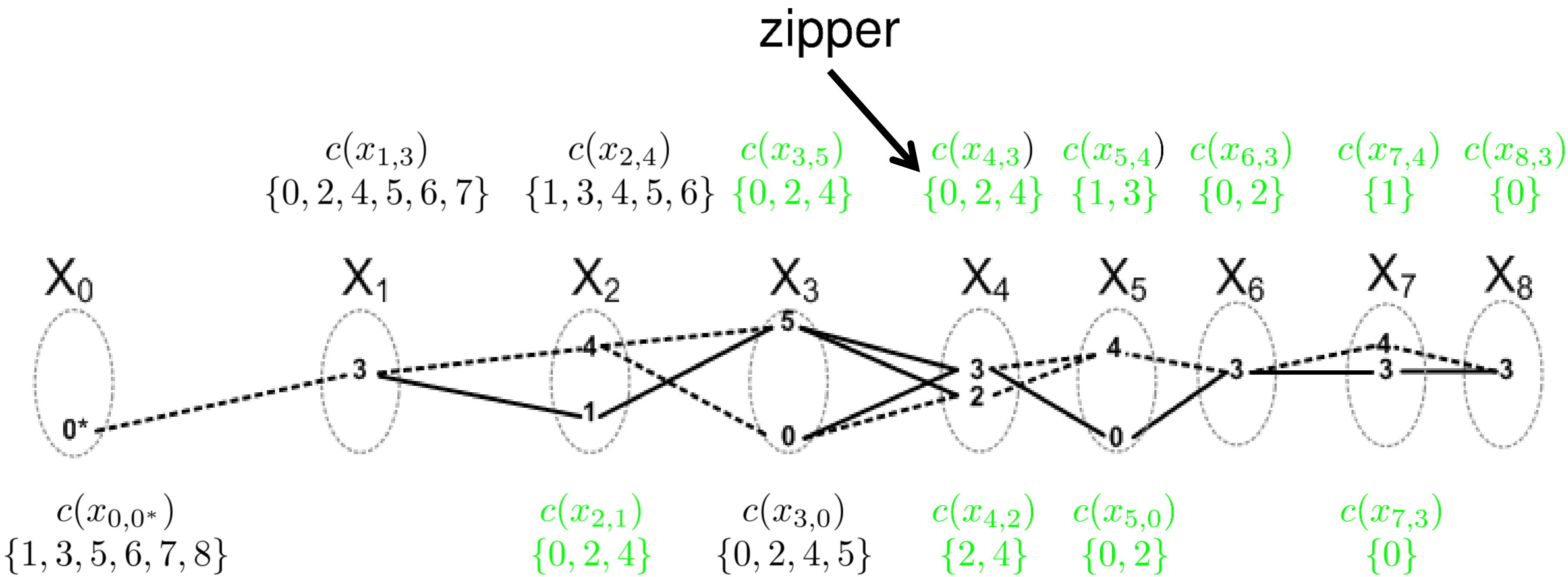
New DC propagator



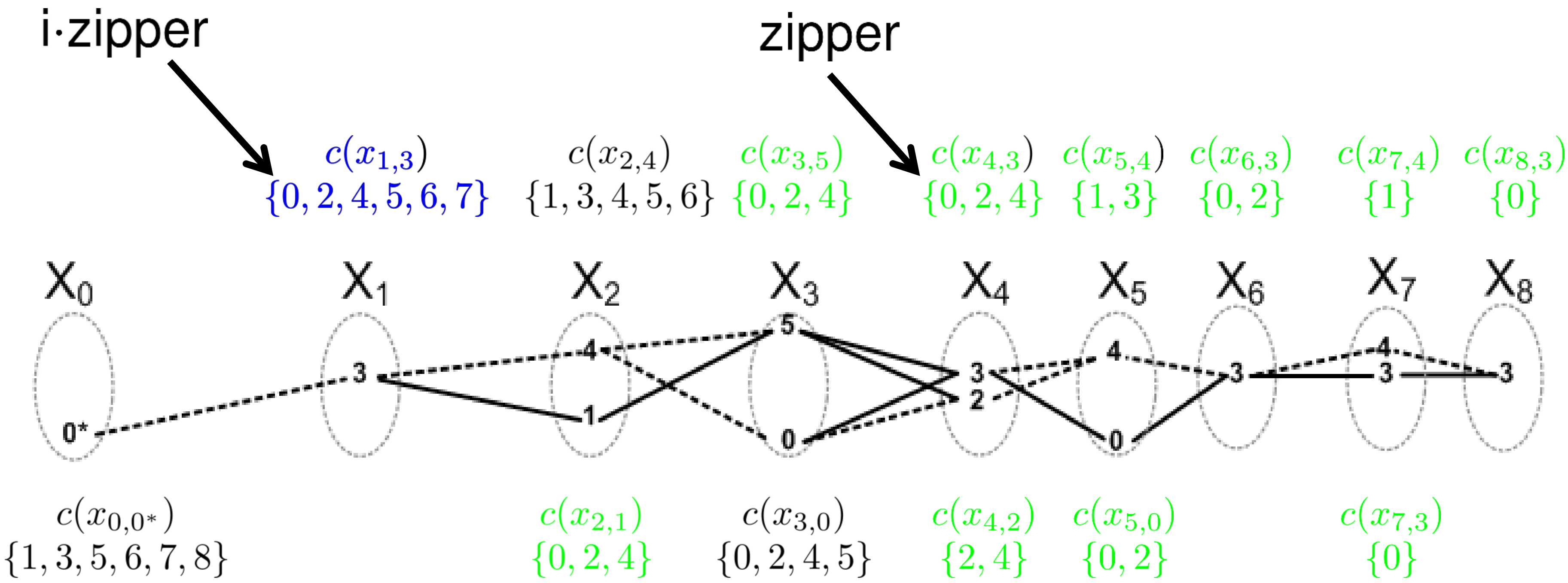
Uniqueness


zipper
$c(x_{1,3})$ $\{0,2,4,5,6,7\}$
$c(x_{2,4})$ $\{1,3,4,5,6\}$
$c(x_{3,5})$ $\{0,2,4\}$
$c(x_{4,3})$ $\{0,2,4\}$
$c(x_{5,4})$ $\{1,3\}$
$c(x_{6,3})$ $\{0,2\}$
$c(x_{7,4})$ $\{1\}$
$c(x_{8,3})$ $\{0\}$
X_0 X_1 X_2 X_3 X_4 X_5 X_6 X_7 X_8
0*
3
4
1
5
0
3
2
4
0
3
4
3
3
$c(x_{0,0^*})$ $\{1,3,5,6,7,8\}$
$c(x_{2,1})$ $\{0,2,4\}$
$c(x_{3,0})$ $\{0,2,4,5\}$
$c(x_{4,2})$ $\{2,4\}$
$c(x_{5,0})$ $\{0,2\}$
$c(x_{7,3})$ $\{0\}$

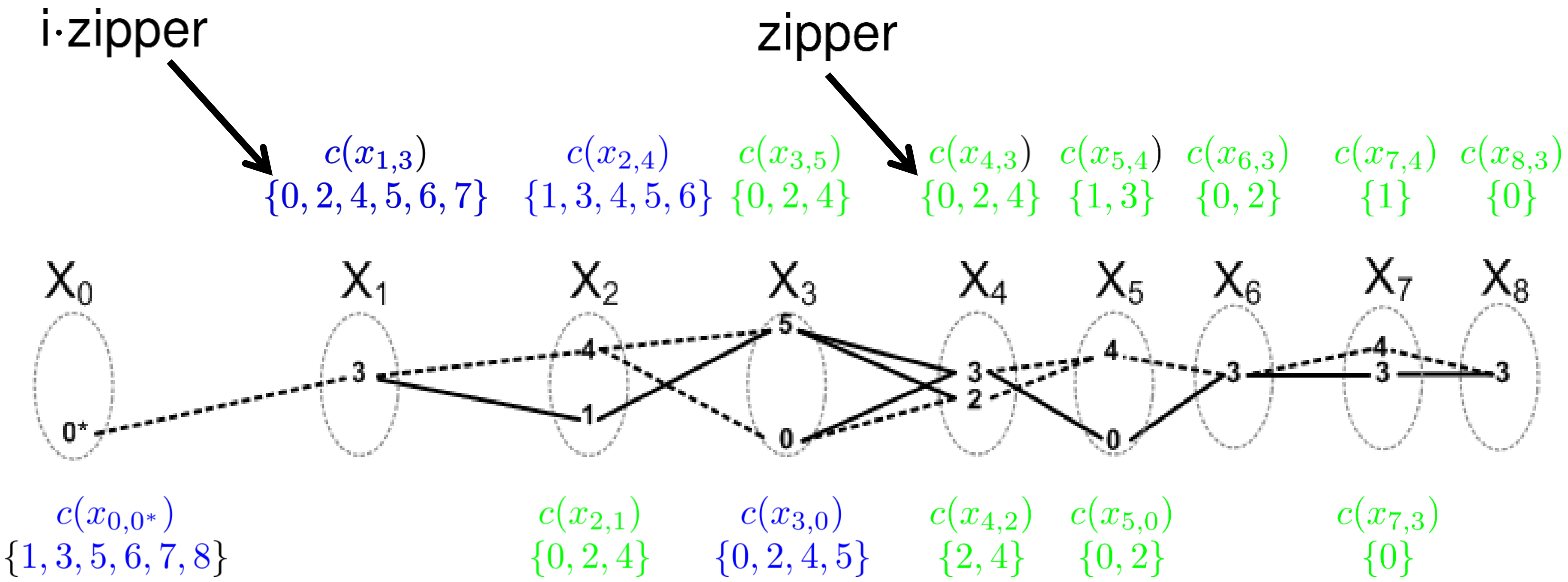
Uniqueness


zipper
$c(x_{1,3})$ $\{0, 2, 4, 5, 6, 7\}$
$c(x_{2,4})$ $\{1, 3, 4, 5, 6\}$
$c(x_{3,5})$ $\{0, 2, 4\}$
$c(x_{4,3})$ $\{0, 2, 4\}$
$c(x_{5,4})$ $\{1, 3\}$
$c(x_{6,3})$ $\{0, 2\}$
$c(x_{7,4})$ $\{1\}$
$c(x_{8,3})$ $\{0\}$
X_0 X_1 X_2 X_3 X_4 X_5 X_6 X_7 X_8
$c(x_{0,0^*})$ $\{1, 3, 5, 6, 7, 8\}$
$c(x_{2,1})$ $\{0, 2, 4\}$
$c(x_{3,0})$ $\{0, 2, 4, 5\}$
$c(x_{4,2})$ $\{2, 4\}$
$c(x_{5,0})$ $\{0, 2\}$
$c(x_{7,3})$ $\{0\}$

Uniqueness


i·zipper
zipper
$c(x_{1,3})$ $\{0,2,4,5,6,7\}$
$c(x_{2,4})$ $\{1,3,4,5,6\}$
$c(x_{3,5})$ $\{0,2,4\}$
$c(x_{4,3})$ $\{0,2,4\}$
$c(x_{5,4})$ $\{1,3\}$
$c(x_{6,3})$ $\{0,2\}$
$c(x_{7,4})$ $\{1\}$
$c(x_{8,3})$ $\{0\}$
X_0 X_1 X_2 X_3 X_4 X_5 X_6 X_7 X_8
$c(x_{0,0^*})$ $\{1,3,5,6,7,8\}$
$c(x_{2,1})$ $\{0,2,4\}$
$c(x_{3,0})$ $\{0,2,4,5\}$
$c(x_{4,2})$ $\{2,4\}$
$c(x_{5,0})$ $\{0,2\}$
$c(x_{7,3})$ $\{0\}$

Uniqueness


i·zipper
zipper
$c(x_{1,3})$ $\{0,2,4,5,6,7\}$
$c(x_{2,4})$ $\{1,3,4,5,6\}$
$c(x_{3,5})$ $\{0,2,4\}$
$c(x_{4,3})$ $\{0,2,4\}$
$c(x_{5,4})$ $\{1,3\}$
$c(x_{6,3})$ $\{0,2\}$
$c(x_{7,4})$ $\{1\}$
$c(x_{8,3})$ $\{0\}$
X_0 X_1 X_2 X_3 X_4 X_5 X_6 X_7 X_8
$c(x_{0,0^*})$ $\{1,3,5,6,7,8\}$
$c(x_{2,1})$ $\{0,2,4\}$
$c(x_{3,0})$ $\{0,2,4,5\}$
$c(x_{4,2})$ $\{2,4\}$
$c(x_{5,0})$ $\{0,2\}$
$c(x_{7,3})$ $\{0\}$

NICTA

Structure (Bounded holes)

The number of holes in an i·zipper is bounded.

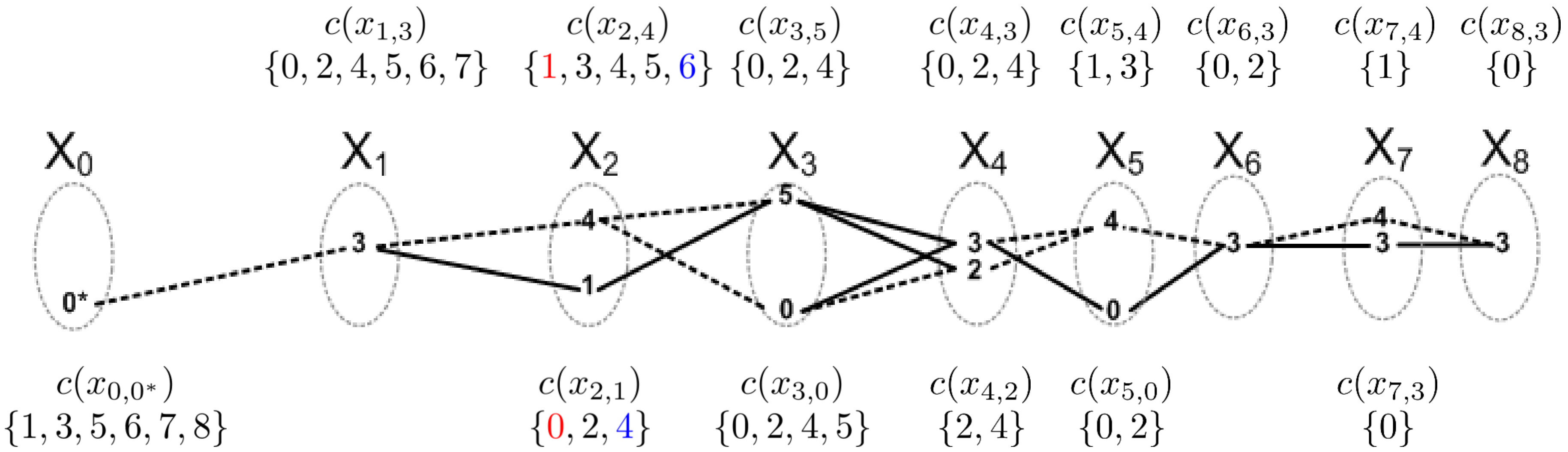
Structure (Bounded holes)

i·zipper: $\{2, 4\} \cup [4, 11] \cup \{11, 13, 15\}$

Structure (Closeness)

Bounds of $c(x_{i,j})$ and $c(x_{i,k})$ are close to each other.

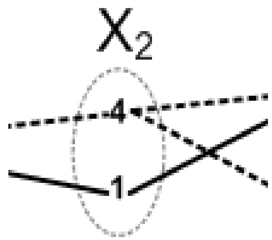
Structure (Closeness)



Structure (Closeness)

$c(x_{2,4})$

$\{1, 3, 4, 5, 6\}$



$c(x_{2,1})$

$\{0, 2, 4\}$

More properties

and many more properties...

NICTA

How did we know?

zipper: $\{2, 4, 6, 8\}$.

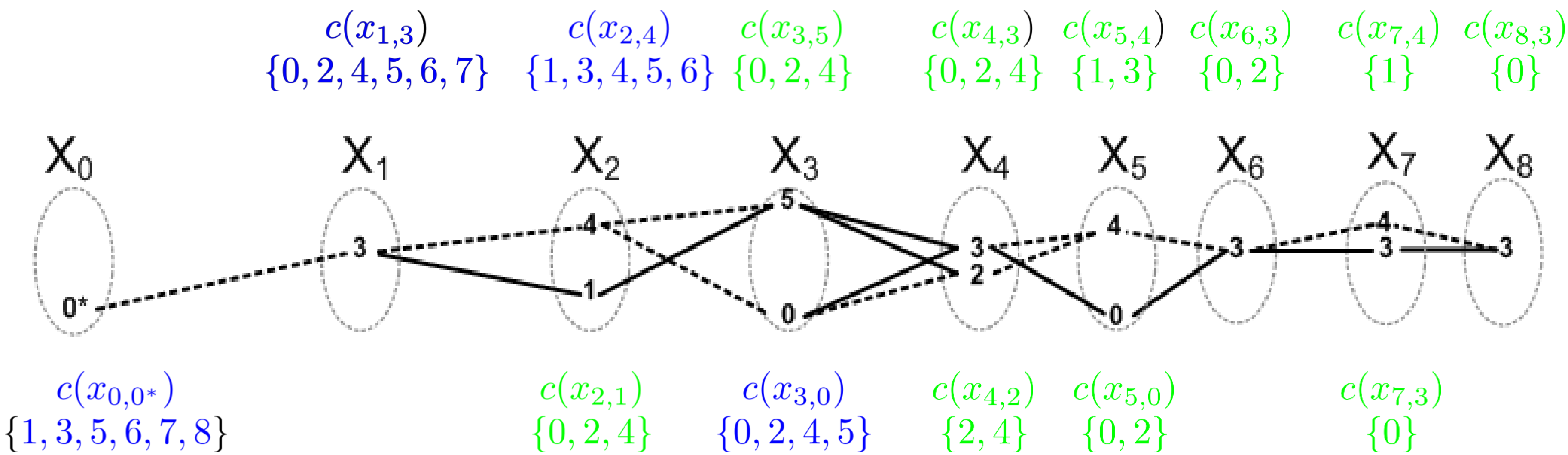
i·zipper: $\{2, 4\} \cup [4, 11] \cup \{11, 13, 15\}$

SeqBin

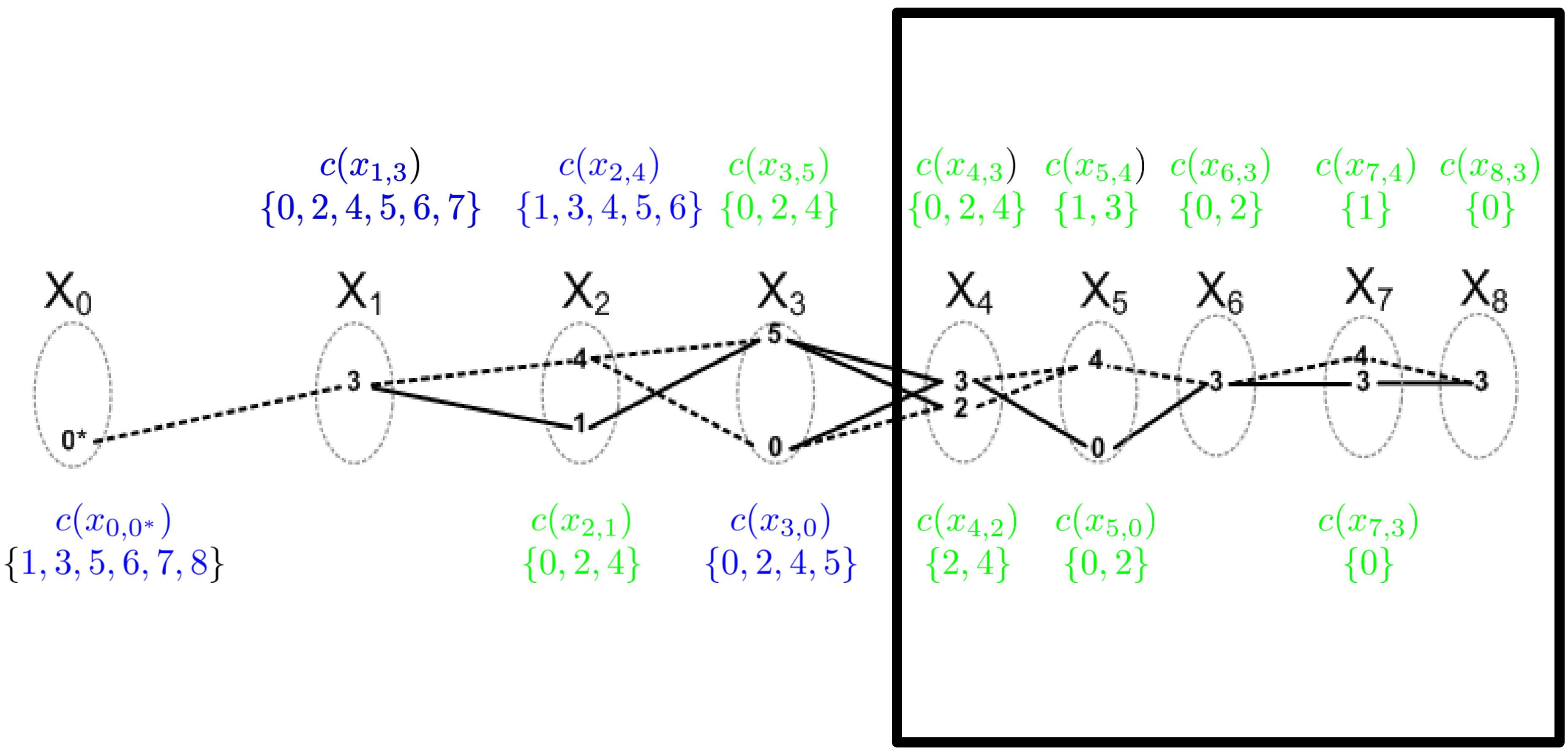
Global structure of cost

NICTA

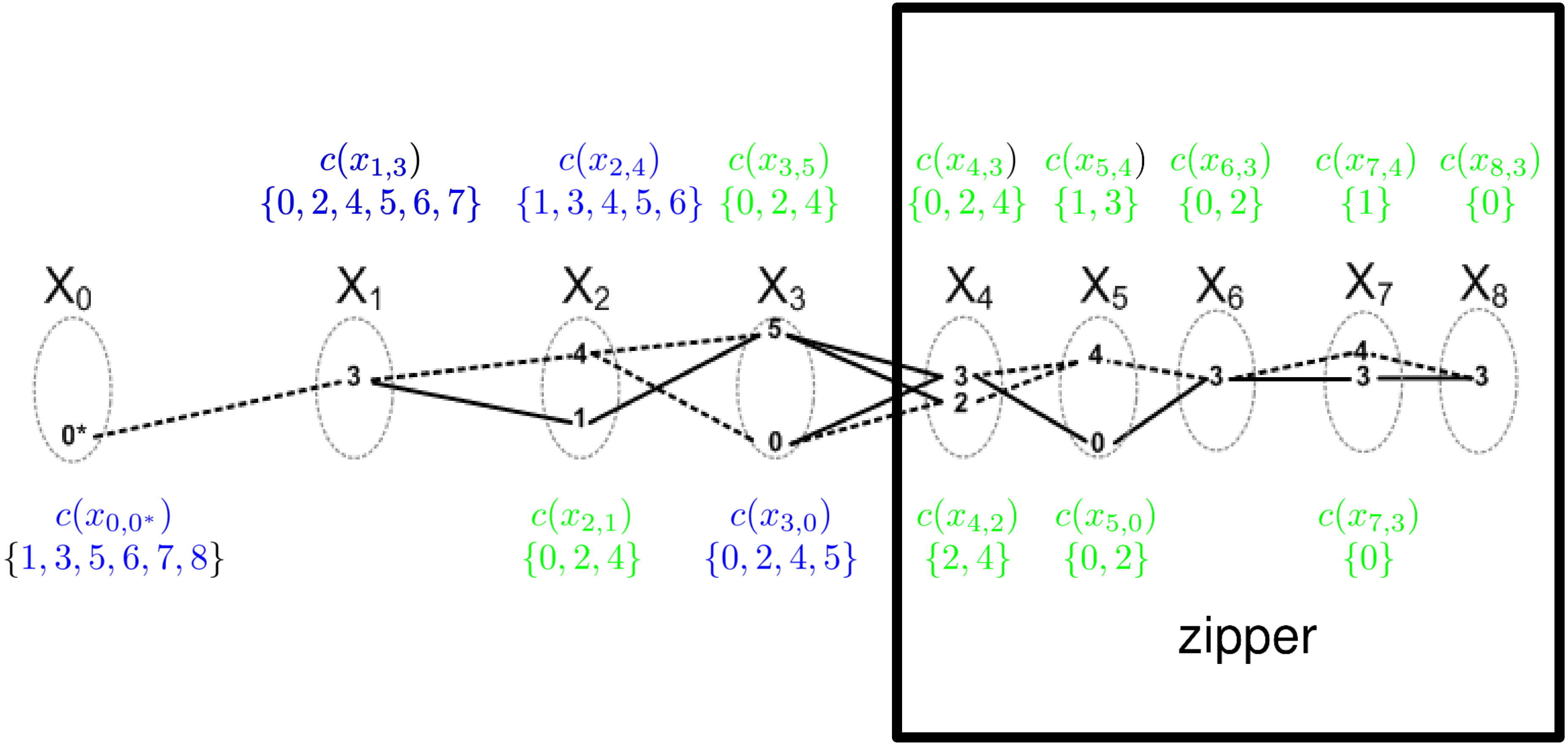
Blocks



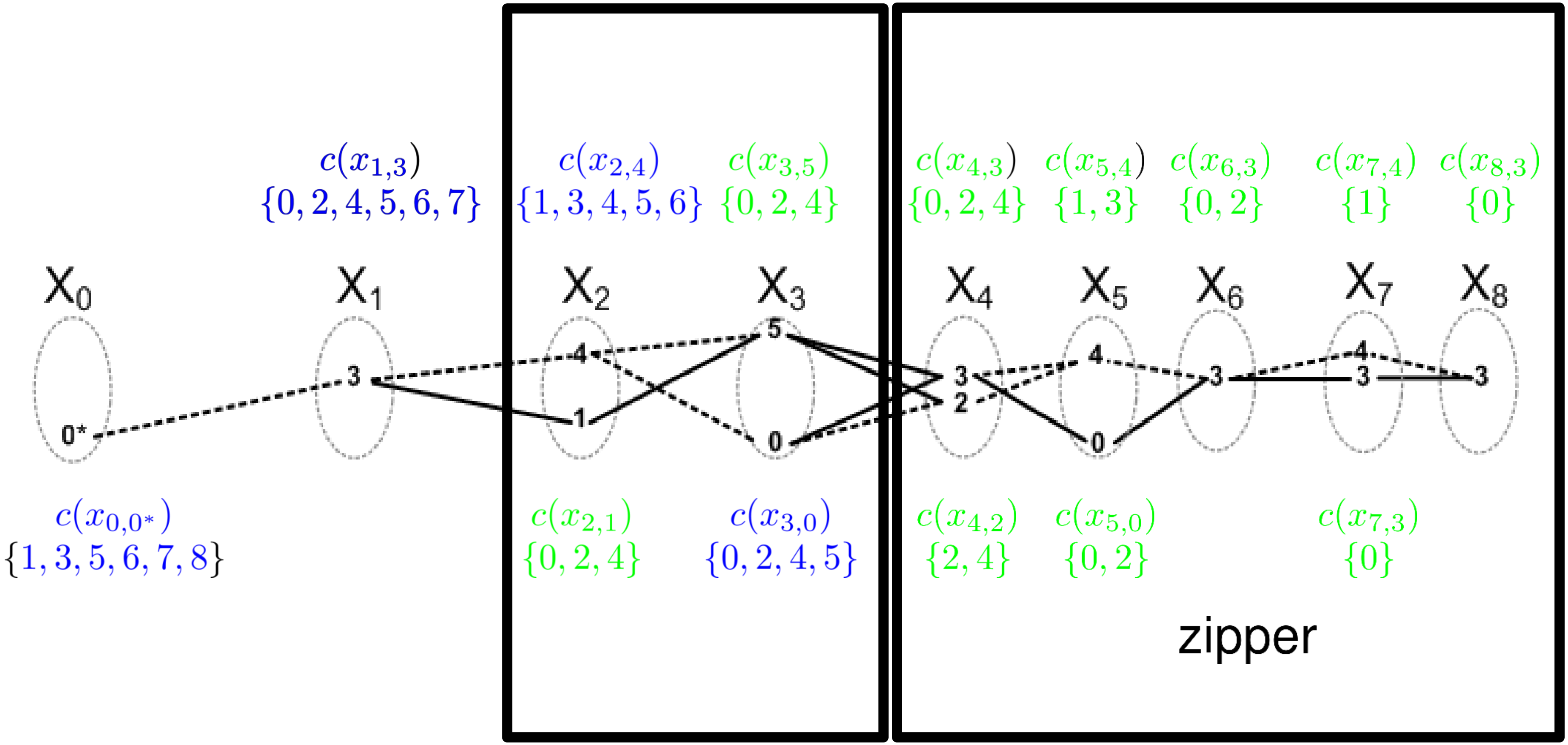
Blocks


$c(x_{1,3})$
$\{0,2,4,5,6,7\}$
$c(x_{2,4})$
$\{1,3,4,5,6\}$
$c(x_{3,5})$
$\{0,2,4\}$
$c(x_{4,3})$
$\{0,2,4\}$
$c(x_{5,4})$
$\{1,3\}$
$c(x_{6,3})$
$\{0,2\}$
$c(x_{7,4})$
$\{1\}$
$c(x_{8,3})$
$\{0\}$
X_0
X_1
X_2
X_3
X_4
X_5
X_6
X_7
X_8
0*
3
4
1
5
0
3
2
4
0
3
4
3
3
$c(x_{0,0^*})$
$\{1,3,5,6,7,8\}$
$c(x_{2,1})$
$\{0,2,4\}$
$c(x_{3,0})$
$\{0,2,4,5\}$
$c(x_{4,2})$
$\{2,4\}$
$c(x_{5,0})$
$\{0,2\}$
$c(x_{7,3})$
$\{0\}$

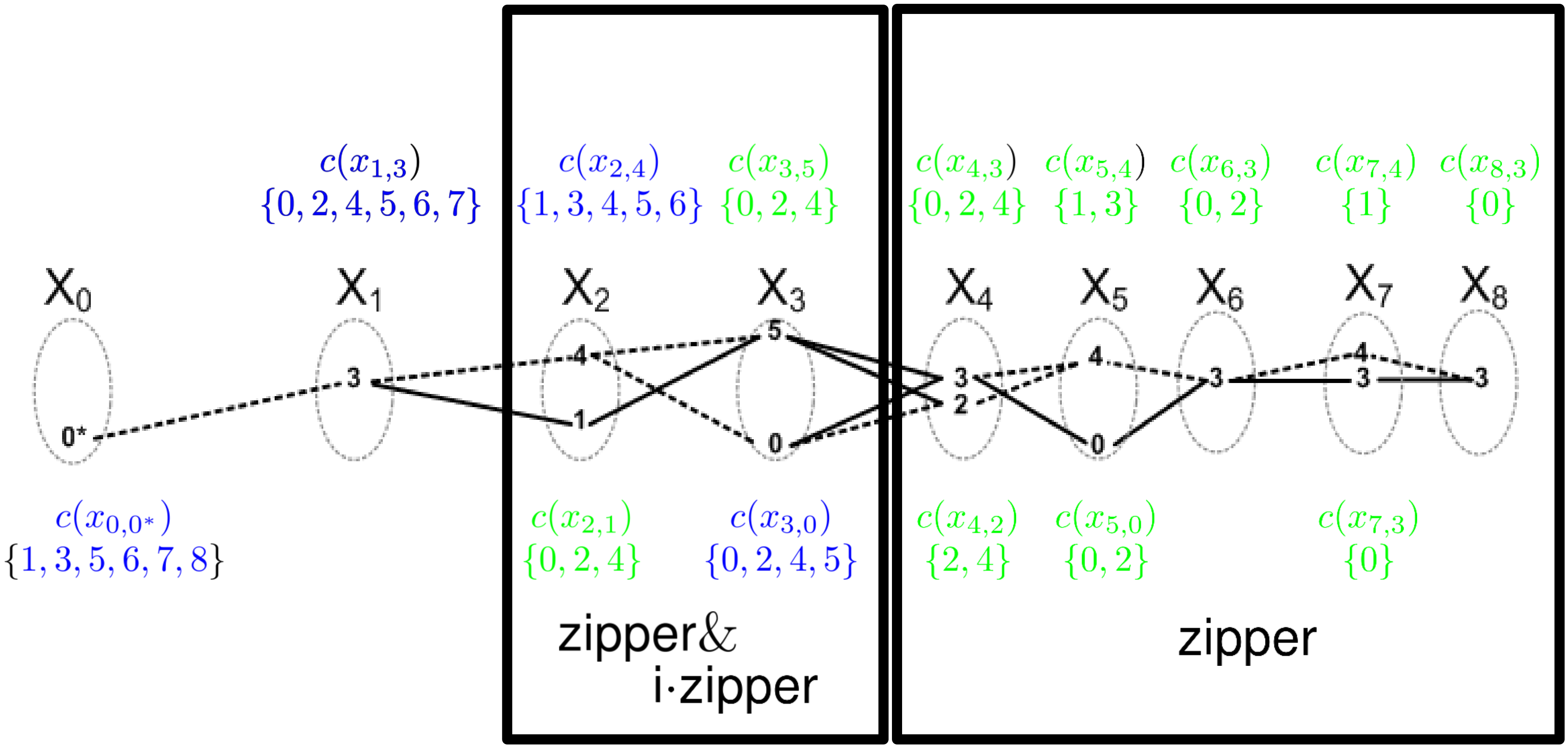
Blocks


$c(x_{1,3})$ $\{0,2,4,5,6,7\}$
$c(x_{2,4})$ $\{1,3,4,5,6\}$
$c(x_{3,5})$ $\{0,2,4\}$
$c(x_{4,3})$ $\{0,2,4\}$
$c(x_{5,4})$ $\{1,3\}$
$c(x_{6,3})$ $\{0,2\}$
$c(x_{7,4})$ $\{1\}$
$c(x_{8,3})$ $\{0\}$
X0
X1
X2
X3
X4
X5
X6
X7
X8
0*
3
4
1
5
0
3
2
4
0
3
4
3
3
$c(x_{0,0^*})$ $\{1,3,5,6,7,8\}$
$c(x_{2,1})$ $\{0,2,4\}$
$c(x_{3,0})$ $\{0,2,4,5\}$
$c(x_{4,2})$ $\{2,4\}$
$c(x_{5,0})$ $\{0,2\}$
$c(x_{7,3})$ $\{0\}$
zipper

Blocks


$c(x_{1,3})$
$\{0,2,4,5,6,7\}$
$c(x_{2,4})$
$\{1,3,4,5,6\}$
$c(x_{3,5})$
$\{0,2,4\}$
$c(x_{4,3})$
$\{0,2,4\}$
$c(x_{5,4})$
$\{1,3\}$
$c(x_{6,3})$
$\{0,2\}$
$c(x_{7,4})$
$\{1\}$
$c(x_{8,3})$
$\{0\}$
X_0 X_1 X_2 X_3 X_4 X_5 X_6 X_7 X_8
0*
3
4
1
5
0
3
2
4
0
3
4
3
3
$c(x_{0,0^*})$
$\{1,3,5,6,7,8\}$
$c(x_{2,1})$
$\{0,2,4\}$
$c(x_{3,0})$
$\{0,2,4,5\}$
$c(x_{4,2})$
$\{2,4\}$
$c(x_{5,0})$
$\{0,2\}$
$c(x_{7,3})$
$\{0\}$
zipper

Blocks


$c(x_{1,3})$
$\{0, 2, 4, 5, 6, 7\}$
$c(x_{2,4})$
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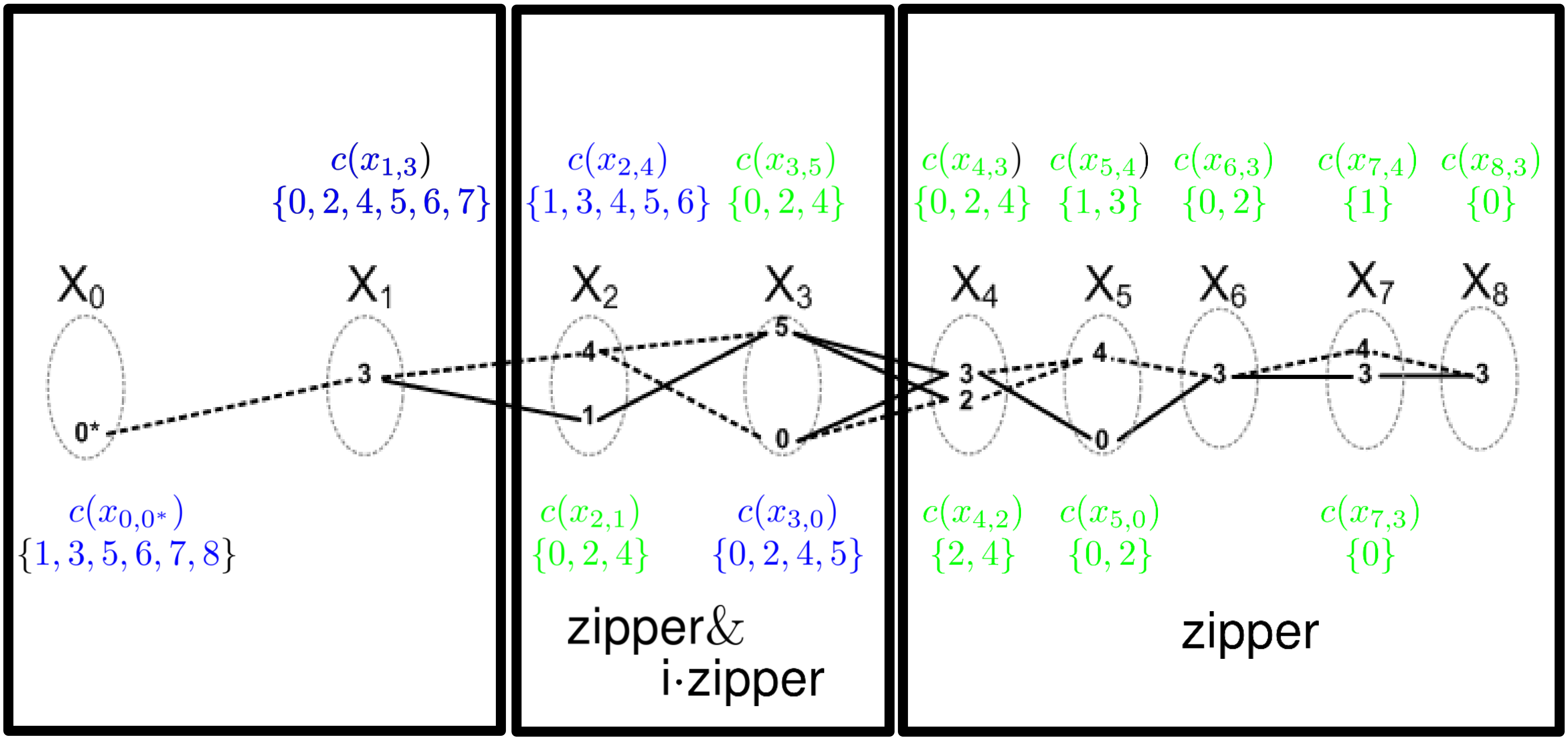
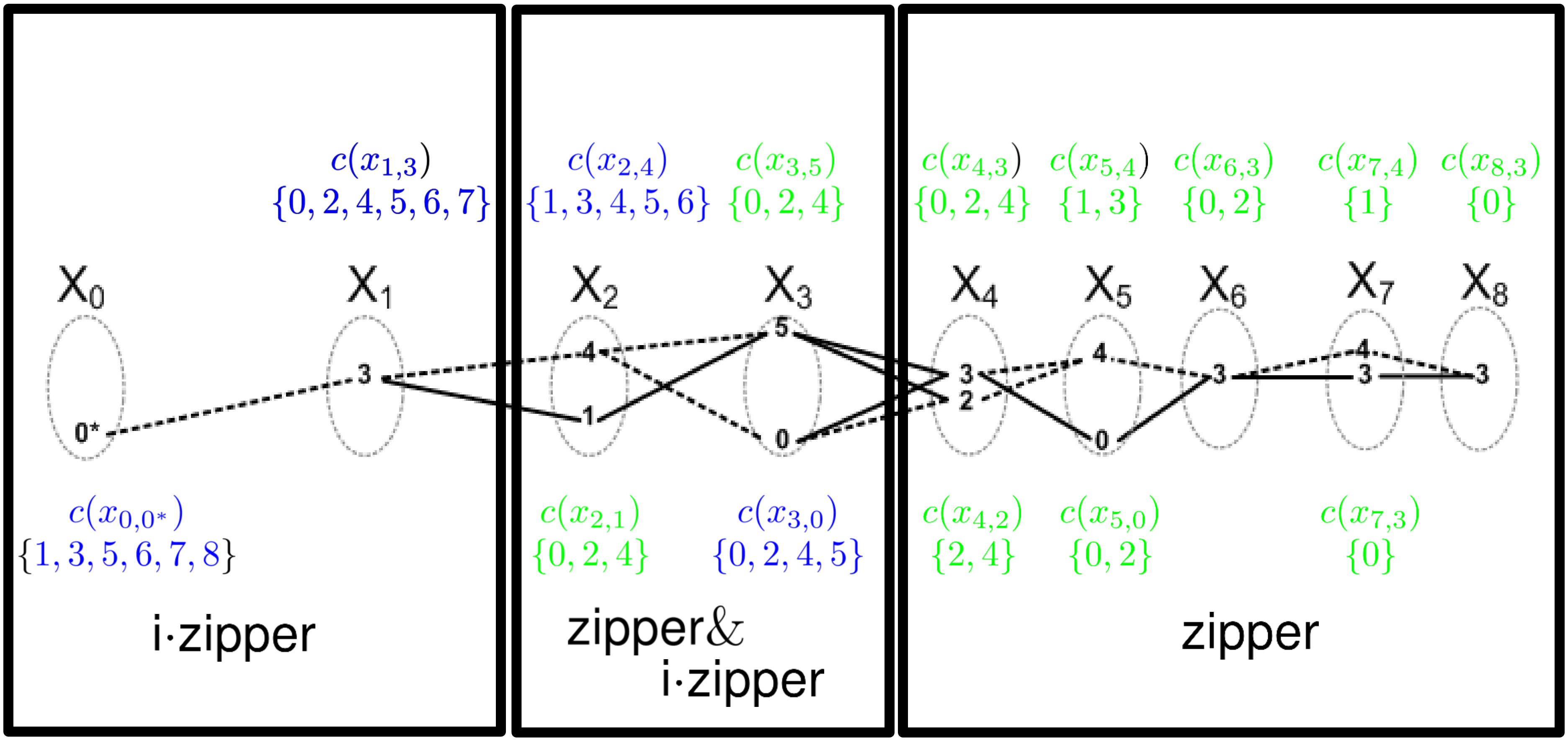
$c(x_{4,3})$
$\{0, 2, 4\}$
$c(x_{5,4})$
$\{1, 3\}$
$c(x_{6,3})$
$\{0, 2\}$
$c(x_{7,4})$
$\{1\}$
$c(x_{8,3})$
$\{0\}$
X_0
X_1
X_2
X_3
X_4
X_5
X_6
X_7
X_8
0*
3
4
1
5
0
3
2
4
0
3
4
3
3
$c(x_{0,0^*})$
$\{1, 3, 5, 6, 7, 8\}$
$c(x_{2,1})$
$\{0, 2, 4\}$
$c(x_{3,0})$
$\{0, 2, 4, 5\}$
$c(x_{4,2})$
$\{2, 4\}$
$c(x_{5,0})$
$\{0, 2\}$
$c(x_{7,3})$
$\{0\}$
zipper&
i·zipper
zipper

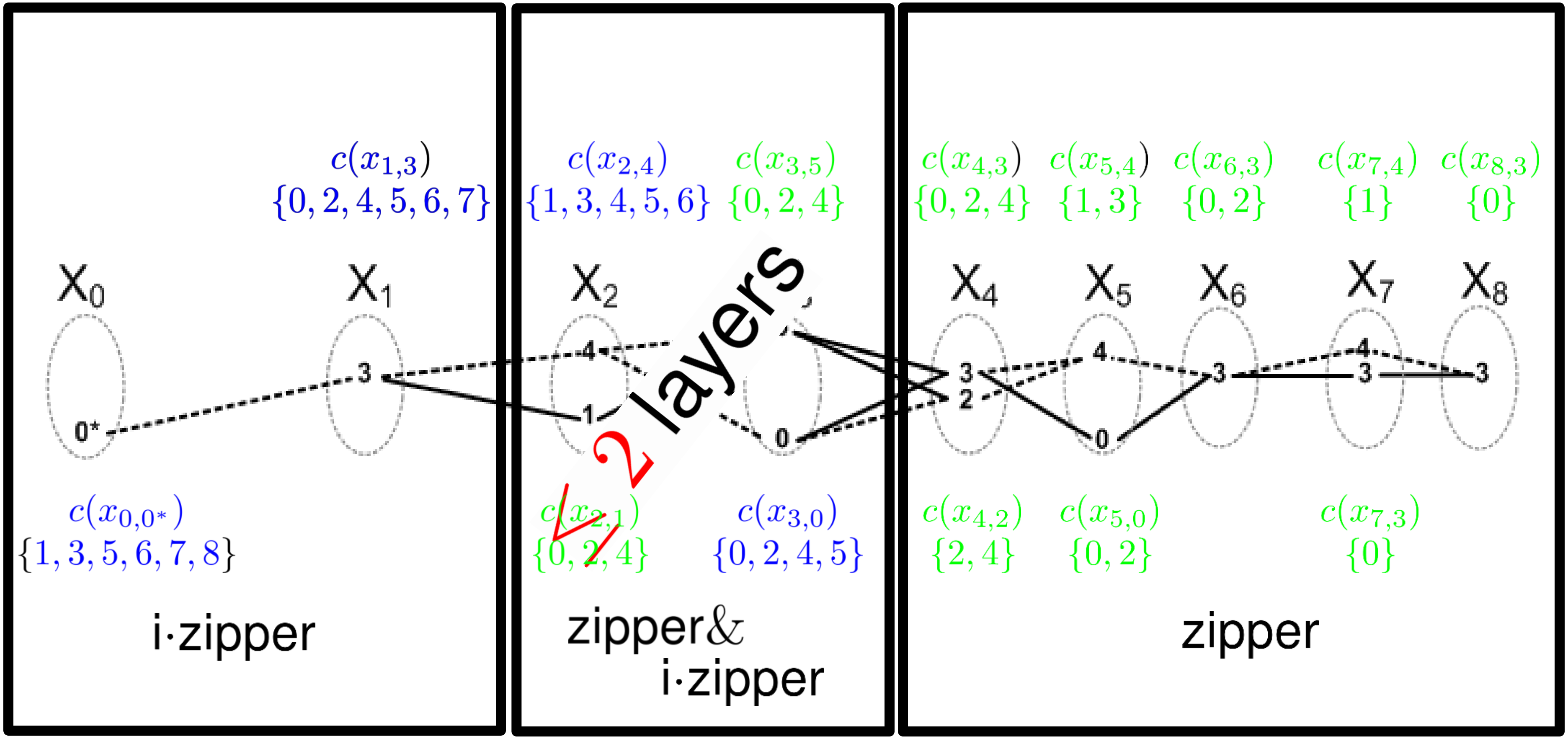
Blocks


$c(x_{1,3})$
$\{0,2,4,5,6,7\}$
$c(x_{2,4})$
$\{1,3,4,5,6\}$
$c(x_{3,5})$
$\{0,2,4\}$
$c(x_{4,3})$
$\{0,2,4\}$
$c(x_{5,4})$
$\{1,3\}$
$c(x_{6,3})$
$\{0,2\}$
$c(x_{7,4})$
$\{1\}$
$c(x_{8,3})$
$\{0\}$
X0 X1 X2 X3 X4 X5 X6 X7 X8
$c(x_{0,0^*})$
$\{1,3,5,6,7,8\}$
$c(x_{2,1})$
$\{0,2,4\}$
$c(x_{3,0})$
$\{0,2,4,5\}$
$c(x_{4,2})$
$\{2,4\}$
$c(x_{5,0})$
$\{0,2\}$
$c(x_{7,3})$
$\{0\}$
zipper&
i·zipper
zipper

Blocks


$c(x_{1,3})$
$\{0,2,4,5,6,7\}$
$c(x_{2,4})$
$\{1,3,4,5,6\}$
$c(x_{3,5})$
$\{0,2,4\}$
$c(x_{4,3})$
$\{0,2,4\}$
$c(x_{5,4})$
$\{1,3\}$
$c(x_{6,3})$
$\{0,2\}$
$c(x_{7,4})$
$\{1\}$
$c(x_{8,3})$
$\{0\}$
X_0
X_1
X_2
X_3
X_4
X_5
X_6
X_7
X_8
$c(x_{0,0^*})$
$\{1,3,5,6,7,8\}$
$c(x_{2,1})$
$\{0,2,4\}$
$c(x_{3,0})$
$\{0,2,4,5\}$
$c(x_{4,2})$
$\{2,4\}$
$c(x_{5,0})$
$\{0,2\}$
$c(x_{7,3})$
$\{0\}$
i·zipper
zipper&
i·zipper
zipper

Blocks


$c(x_{1,3})$
$\{0,2,4,5,6,7\}$
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$\{1,3,4,5,6\}$
$c(x_{3,5})$
$\{0,2,4\}$
$c(x_{4,3})$
$\{0,2,4\}$
$c(x_{5,4})$
$\{1,3\}$
$c(x_{6,3})$
$\{0,2\}$
$c(x_{7,4})$
$\{1\}$
$c(x_{8,3})$
$\{0\}$
X0 X1 X2 X4 X5 X6 X7 X8
2 layers
$c(x_{0,0^*})$
$\{1,3,5,6,7,8\}$
$c(x_{2,1})$
$\{0,2,4\}$
$c(x_{3,0})$
$\{0,2,4,5\}$
$c(x_{4,2})$
$\{2,4\}$
$c(x_{5,0})$
$\{0,2\}$
$c(x_{7,3})$
$\{0\}$
i·zipper
zipper&
i·zipper
zipper

Conclusions

NICTA

Contributions

If B is monotone then DC propagator runs in $O(nd^2)$

Contributions

If B is monotone then DC propagator runs in $O(nd^2)$ 

If B is monotone and C is convex

then DC propagator runs in $O(nd)$ 

Thank you!